

# New directions in atomic PNC

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## Abstract

A preliminary all-order calculation of the spin-dependent (PNC) amplitude in Cs is carried out in an attempt to understand discrepancies in weak nucleon-nucleon coupling constants derived from atomic and nuclear experiments [W.C. Haxton and C.E. Wieman, *Ann. Rev. Nucl. Part. Sci.* **51**, 261 (2001)]. We find that individual PNC matrix elements significantly change with more complete inclusion of the correlation corrections, whereas changes in the total spin-dependent PNC amplitude are relatively small. Our preliminary value of the anapole coupling constant  $\kappa_a = 0.88(12)$  is only 5% lower than the value used by Haxton and Wieman.

*Key words:* Parity violation, Anapole moment, Hadronic weak interactions, All-order method, Perturbation theory, Cesium

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Important goals of high-precision atomic parity nonconservation (PNC) studies are to search for new physics beyond the standard model of the electroweak interaction by precise evaluation of the weak charge  $Q_W$  and to probe parity violation in the nucleus by evaluation of the nuclear anapole moment. Atomic PNC measurements have been completed in Cs (0.35 % accuracy [2]), Tl (1.7 %), Bi (2 %), Pb (1.2 %), as well as Sm and Dy. Several atomic PNC experiments (in Fr, Ba<sup>+</sup>, Ra<sup>+</sup>, and Yb) are currently in progress. Atomic-physics tests of the standard model provide important constraints on possible extensions of the standard model. A recent analysis [3] of parity-violating electron-nucleus scattering measurements combined with atomic PNC measurements placed tight constraints on the weak neutral-current lepton-quark interactions at low energy, improving the lower bound on the scale of relevant new physics to  $\sim 1$  TeV.

The precise measurement of PNC amplitudes in Cs [2] led to an experimental value of the small contribution from the nuclear-spin dependent PNC accurate to 14%. The

constraints on weak nucleon-nucleon coupling constants derived from this experiment and calculations in Cs were found to be inconsistent with constraints from deep inelastic scattering and other nuclear experiments [1,4], which was based on older calculations of the spin-dependent PNC amplitude [5,6]. In Ref. [5], the PNC amplitude was evaluated using a semi-empirical method, and in Ref. [6] the relativistic Hartree-Fock method and Brueckner orbitals were used. The spin-dependent part of the PNC amplitude was also calculated in the Dirac-Fock approximation in Ref. [7]. We have demonstrated in Ref. [8] that Dirac-Fock and random-phase approximation (RPA) values of spin-dependent PNC amplitude differ by as much as 25%. Therefore, it is imperative to include correlation corrections as completely as possible in the calculation of the spin-dependent PNC amplitude to ensure an accurate analysis of the Boulder experiment [2]. In this work, we carried out a high-precision calculation of spin-dependent PNC amplitude in Cs using the relativistic all-order method [9] in attempt to understand the above mentioned discrepancies of weak nucleon-nucleon coupling constants derived from atomic and nuclear experiments.

The spin-dependent contribution to the PNC amplitude has three distinct sources: the nuclear anapole moment, the  $Z$  exchange interaction from nucleon axial-vector currents ( $A_n V_e$ ), and the combined action of the hyperfine interaction and spin-independent  $Z$  exchange from nucleon vector ( $V_n A_e$ ) currents. The anapole moment contribution strongly dominates. The first two of the above-mentioned interactions can be represented by the same Hamiltonian

$$H^{(i)} = \frac{G}{\sqrt{2}} \kappa_i \boldsymbol{\alpha} \cdot \mathbf{I} \rho(r), \quad (1)$$

where the subscript  $i = a, 2$  refers to the anapole moment and the axial-vector contributions, respectively. In Eq. (1),  $G$  is the universal weak coupling constant,  $\mathbf{I}$  is the nuclear spin, and  $\rho(r)$  is a normalized density function. The dimensionless constant  $\kappa_a$  is used to characterize anapole moment [4]. The constant  $\kappa_2 = 0.0140$  was calculated in Ref. [4]. The total spin-dependent PNC contribution to the electric-dipole matrix element  $\langle 7sF_F \| z \| 6sF_I \rangle$  is given by [8]

$$\langle 7sF_F \| z \| 6sF_I \rangle_{\text{PNC}} = (\kappa_a + \kappa_2 + \kappa_{\text{hf}}) \langle 7sF_F \| z \| 6sF_I \rangle^{(2,a)}. \quad (2)$$

The constant  $\kappa_{\text{hf}}$  that characterizes the third contribution, combined hyperfine and the spin-independent  $Z$  exchange interactions, was calculated using RPA to be  $\kappa_{\text{hf}} = 0.0049$  in Ref. [8].

In this work, we calculated the values of the PNC matrix elements using an all-order method in which all single and double (SD) excitations of the Dirac-Fock wave functions are included to all orders of perturbation theory. The resulting spin-dependent PNC corrections are calculated using the sum over state approach described in Ref. [7]. This all-order method and its applications are discussed in the review [9] and references therein. Preliminary all-order results are listed in Table 1 together with the DF and RPA results of Ref. [8]. We used experimental values of the energies and “best set” data [10] for electric-dipole matrix elements that combines experimental and theoretical values. Combining the all-order results of Table 1 with the experimental value of  $\Delta[\text{Im}(E_{PNC})/\beta]_{34-43} = -0.0077(11)$  mV/cm from Ref. [2] and  $\beta = 27.02(8)a_0^3$  [11], we obtain  $\kappa = \kappa_a + \kappa_2 + \kappa_{\text{hf}} = 0.107(16)$  that is rather close to the previous value of 0.112(16) from Ref. [12] used in the analysis of Haxton and Wieman [1]. We note that the uncertainty in both values is the experimental one. (The theoretical uncertainty is negligible in comparison to the

Table 1

Absolute values of the spin-dependent PNC reduced matrix elements  $\langle 7sF_F || z || 6sF_I \rangle^{(2)}$  calculated using DF, RPA, and SD all-order methods (in a.u.). The DF and RPA results have been published previously in Ref. [8]. The numbers in brackets represent powers of 10.

$F_F - F_I$	DF	RPA	All-order
3 - 4	5.481[-12]	7.299[-12]	7.948[-12]
4 - 3	4.746[-12]	6.432[-12]	7.057[-12]

experimental uncertainty.) The resulting value  $\kappa_a = 0.88(16)$  is obtained by subtracting the value of  $\kappa_2$  listed above and the value of  $\kappa_{\text{hf}}$  from Ref. [8] from the experimental value of  $\kappa$ . While our result is 5% lower than the value  $\kappa_a = 0.92(16)$  used in Ref. [1], it does not change the conclusion of that work.

We have also calculated the parity nonconserving  $7s - 6d$  amplitude  $E1_{\text{PNC}}$  in  $\text{Ra}^+$  needed for future analysis of the ongoing experimental effort in KVI, Groningen, using the high-precision relativistic all-order method. A detailed study of the uncertainty of the PNC amplitude is carried out; additional calculations are performed to estimate some of the missing correlation corrections. A number of parity-conserving quantities, such as energy levels, transition matrix elements for the E1, M1, and E2 transitions, lifetimes, branching ratios, quadrupole moments, ground state dipole and quadrupole polarizabilities, and magnetic-dipole hyperfine constants have been also calculated [13].

Finally, we are currently working on the development of a novel method for precision calculation of properties of atomic systems with a few valence electrons [14] for the evaluation of the PNC amplitudes of more complicated atomic systems, such as Yb, Pb, Bi, and Tl where the theoretical accuracy is currently insufficient for accurate analyses of the experimental results. This method combines the relativistic all-order (coupled-cluster) approach used in our Cs and  $\text{Ra}^+$  with the configuration interaction (CI) approach. In the CI + all-order approach, core excitations are incorporated in the CI method by constructing an effective Hamiltonian using fully converged all-order excitations coefficients.

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