

GUT inflation and proton decay after WMAP5 data

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We employ Coleman-Weinberg and Higgs potentials to implement inflation in nonsupersymmetric grand unified theories (GUTs) such as SU(5) and SO(10). To realize a scalar spectral index close to 0.96, as indicated by the most recent WMAP 5-year analysis, the energy scale of observable inflation turns out to be of order 10^{16} GeV. This implies a GUT symmetry breaking scale of similar magnitude and proton lifetime of order 10^{34} – 10^{38} years. In some SO(10) models with axion dark matter, the scalar leptoquark boson exchange leads to proton decay with a lifetime of order 10^{34} – 10^{35} years.

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I. INTRODUCTION

An inflationary scenario [1] may be termed successful if the following conditions are satisfied:

- (1) The number of e-foldings is sufficient to resolve the horizon and flatness problems.
- (2) The predicted temperature anisotropies, scalar spectral index n_s , tensor-to-scalar ratio r , etc. are in agreement with the observations.
- (3) Following inflation, there exists a mechanism for explaining the observed baryon asymmetry.
- (4) The number density of superheavy magnetic monopoles in grand unified theories (GUTs) is suitably suppressed to agree with the upper bounds on their primordial abundance.
- (5) The model should offer a plausible cold dark matter candidate.

It was shown a long time ago [2–4] that nonsupersymmetric GUT inflation can be realized by employing a Coleman-Weinberg (CW) type potential for a scalar inflaton field which must be a GUT singlet. The singlet condition ensures that radiative corrections from the gauge sector do not spoil the desired inflationary potential. The monopole problem is resolved by requiring that the GUT symmetry breaking, associated with monopole production, takes place during inflation. A suitable coupling between the GUT symmetry breaking field Σ and the inflaton field ϕ is thus needed. This coupling also plays a crucial role in generating an appropriate CW inflationary potential for ϕ .

The most recent Wilkinson Microwave Anisotropy Probe (WMAP) 5-year [5] analyses seem to favor a scalar spectral index centered around 0.96. Within the CW framework this favors an energy scale for inflation which is close to $(1\text{--}2) \times 10^{16}$ GeV [6]. This result, as we will show here, has far reaching implications for the symmetry breaking scale of the underlying GUT. In our CW examples below, primarily based on SO(10), the superheavy gauge bosons which mediate proton decay are typically a factor 2–4 larger in mass than the energy scale of inflation preferred by WMAP5. To phrase things somewhat differently, while the most recent proton lifetime limits from Super-K

($\tau(p \rightarrow \pi^0 e^+) > 8.2 \times 10^{33}$ years [7]) require that $M_X \geq 4 \times 10^{15}$ GeV [8], the limits on M_X from WMAP seem to favor a value which is up to a factor 10 larger. We thus estimate a proton lifetime of around 10^{34} – 10^{38} years. Proton lifetime estimates of order 10^{34} – 10^{35} years are achieved in some SO(10) axion models which contain scalar leptoquarks (3, 1, $-2/3$) with an intermediate mass of order 10^{12} GeV.

As an alternative to the CW potential, we will also discuss inflation with a Higgs potential [1,9], as well as implications for proton decay. As we will see, such a model predicts values for the scalar spectral index n_s , the tensor-to-scalar ratio r , the scale of vacuum energy, and the proton lifetime that are quite similar to those predicted by the CW model.

II. GUT INFLATION WITH A CW POTENTIAL

To simplify the presentation we begin with a brief summary based on SU(5), following Refs. [2,10]. The model contains an SU(5) singlet (real) scalar ϕ which develops a CW potential from its weak couplings to the adjoint and fundamental Higgs fields Σ and H_5 . The tree-level scalar potential is given by

$$V(\phi, \Sigma, H_5) = \frac{1}{4} a (\text{Tr} \Sigma^2)^2 + \frac{1}{2} b \text{Tr} \Sigma^4 - \alpha (H_5^\dagger H_5) \text{Tr} \Sigma^2 + \frac{\beta}{4} (H_5^\dagger H_5)^2 + \gamma H_5^\dagger \Sigma^2 H_5 + \frac{\lambda_1}{4} \phi^4 - \frac{\lambda_2}{2} \phi^2 \text{Tr} \Sigma^2 + \frac{\lambda_3}{2} \phi^2 H_5^\dagger H_5. \quad (1)$$

The coefficients a , b , α , and β are taken to be of order g^2 , so that most radiative corrections in the (Σ, H_5) sector can be neglected. We assume a somewhat smaller value for the coefficient $|\gamma|$ and take $0 < \lambda_i \ll g^2$ and $\lambda_1 \leq \max(\lambda_2^2, \lambda_3^2)$.

Radiative corrections due to the couplings $\phi^2 \text{Tr} \Sigma^2$ and $\phi^2 H_5^\dagger H_5$ induce a Coleman-Weinberg potential for ϕ , which is given by

$$V(\phi) = A\phi^4 \left(\ln\left(\frac{\phi}{M}\right) + C \right) + V_0, \quad (2)$$

with

$$A = \frac{\lambda_2^2}{16\pi^2} \left(1 + \frac{25g^4}{16\lambda_c^2} + \frac{14b^2}{9\lambda_c^2} \right) \quad (3)$$

and $\lambda_c = a + \frac{7}{15}b$. The SU(5) symmetry is broken to $SU(3)_c \times SU(2)_L \times U(1)_Y$ when Σ acquires a vacuum expectation value (VEV)

$$\langle \Sigma \rangle = \sqrt{\frac{1}{15}} \sigma \cdot \text{diag}(1, 1, 1, -\frac{3}{2}, -\frac{3}{2}), \quad (4)$$

where

$$\sigma^2 = \frac{2\lambda_2}{\lambda_c} \phi^2. \quad (5)$$

It is worth mentioning here that the SU(5) symmetry breaking in our model does not experience any supercooling. As the inflaton field ϕ drives inflation, the term proportional to $-\lambda_2$ in Eq. (1) induces an instability in the potential for the SU(5) adjoint field Σ . Thus, the SU(5) breaking in our model is definitely not of the Coleman-Weinberg type [11]. Indeed, the Hawking temperature $T_H (= \frac{H}{2\pi})$, where H is the Hubble constant) during observable inflation is of order 10^{13} – 10^{14} GeV (from Table I), so that SU(5) remains

weakly interacting before it is spontaneously broken through a second order phase transition.

With Σ given in Eq. (4), the (ϕ, σ) sector of the effective potential can be written as

$$V = \frac{\lambda_c}{16} \sigma^4 - \frac{\lambda_2}{4} \sigma^2 \phi^2 + A\phi^4 \left(\ln\left(\frac{\phi}{M}\right) + C \right) + V_0. \quad (6)$$

Using Eq. (5) and an appropriate choice of the normalization constant C , the effective potential for the inflaton field ϕ can be expressed in the standard form [10]

$$V(\phi) = A\phi^4 \left[\ln\left(\frac{\phi}{M}\right) - \frac{1}{4} \right] + \frac{AM^4}{4}, \quad (7)$$

where $M = \langle \phi \rangle$ denotes the VEV of ϕ at the minimum. Note that the vacuum energy density at the origin is given by $V_0 = \frac{AM^4}{4}$ such that $V(\phi = M) = 0$, and the corresponding minimum in σ is located at $\sigma_0 = \sqrt{\frac{2\lambda_2}{\lambda_c}} M$. The mass of the superheavy gauge bosons X which mediate proton decay is given by

$$M_X = \sqrt{\frac{5}{3}} \frac{g\sigma_0}{2} = \sqrt{\frac{5\lambda_2 g^2}{3\lambda_c A^{1/2}}} V_0^{1/4}. \quad (8)$$

Thus M_X is estimated to be a factor 2–4 larger than the scale of vacuum energy during inflation. This is to be contrasted with the simplest supersymmetric hybrid infla-

TABLE I. Predicted values of various inflationary parameters using the Coleman-Weinberg potential given in Eq. (7). Here we show values which fall inside the WMAP5 2σ bounds (see Fig. 1), omitting values with very low reheate temperature $T_r (< 10^6$ GeV). Note that unless otherwise specified, we use units of $m_p = 1$. Although M , ϕ_0 , and ϕ_e carry trans-Planckian values, the vacuum energy scale during observable inflation is well below m_p .

$V_0^{1/4}$ (GeV)	$V(\phi_0)^{1/4}$ (GeV)	A (10^{-14})	M	ϕ_0	ϕ_e	n_s	r	T_r (GeV)	$\frac{dn_s}{d \ln k} (-10^{-3})$
$2. \times 10^{15}$	2.00×10^{15}	3.7	2.65	0.11	2.05	0.9369	0.000013	1.54×10^8	1.31
$5. \times 10^{15}$	5.00×10^{15}	6.1	5.85	0.65	5.06	0.9375	0.000496	4.14×10^7	1.21
$8. \times 10^{15}$	7.98×10^{15}	7.3	8.95	1.71	8.09	0.9421	0.00322	2.09×10^7	0.982
$1. \times 10^{16}$	9.93×10^{15}	7.2	11.2	2.82	10.3	0.9465	0.00773	1.49×10^7	0.832
1.25×10^{16}	1.23×10^{16}	6.1	14.6	4.91	13.7	0.9525	0.0180	1.05×10^7	0.703
1.5×10^{16}	1.44×10^{16}	4.4	19.0	8.18	18.1	0.9578	0.0341	7.65×10^6	0.652
1.75×10^{16}	1.61×10^{16}	2.8	24.8	13.1	23.9	0.9613	0.0538	5.73×10^6	0.654
$2. \times 10^{16}$	1.74×10^{16}	1.7	32.3	19.9	31.4	0.9630	0.0730	4.40×10^6	0.670
2.25×10^{16}	1.83×10^{16}	0.99	41.4	28.5	40.5	0.9637	0.0889	3.45×10^6	0.686
2.5×10^{16}	1.89×10^{16}	0.61	52.0	38.8	51.1	0.9638	0.101	2.77×10^6	0.697
2.75×10^{16}	1.93×10^{16}	0.39	64.0	50.5	63.0	0.9636	0.110	2.27×10^6	0.704
$3. \times 10^{16}$	1.96×10^{16}	0.26	77.1	63.5	76.2	0.9635	0.117	1.90×10^6	0.710
$4. \times 10^{16}$	2.03×10^{16}	0.072	142	128	141	0.9628	0.133	1.05×10^6	0.724
$4. \times 10^{16}$	2.14×10^{16}	0.047	157	172	159	0.9607	0.165	1.0×10^6	0.743
$3. \times 10^{16}$	2.17×10^{16}	0.13	92.5	108	93.5	0.9600	0.174	1.73×10^6	0.744
2.75×10^{16}	2.19×10^{16}	0.17	79.1	94.3	80.1	0.9597	0.178	2.04×10^6	0.745
2.5×10^{16}	2.20×10^{16}	0.22	66.9	82.2	67.9	0.9592	0.183	2.45×10^6	0.747
2.25×10^{16}	2.22×10^{16}	0.3	55.8	71.3	56.8	0.9587	0.189	2.98×10^6	0.750
$2. \times 10^{16}$	2.24×10^{16}	0.41	45.8	61.5	46.9	0.9580	0.196	3.69×10^6	0.755
1.75×10^{16}	2.26×10^{16}	0.57	37.0	52.9	38.0	0.9570	0.205	4.70×10^6	0.763
1.5×10^{16}	2.30×10^{16}	0.79	29.2	45.4	30.3	0.9557	0.216	6.17×10^6	0.775
1.25×10^{16}	2.33×10^{16}	1.1	22.5	39.1	23.5	0.9539	0.230	8.44×10^6	0.794

tion models in which the corresponding M_X can easily be 1–2 orders of magnitude larger than the scale of vacuum energy during inflation [12].

The inflationary potential in Eq. (7) is typical for the new inflation scenario [13], where inflation takes place near the maximum. However, as shown in Ref. [6], depending on the value of V_0 , the inflaton can have small or large values compared to the Planck scale during observable inflation. In the latter case, observable inflation takes place near the minimum of $V(\phi)$, and the model mimics chaotic inflation [14]. Indeed, we will see from Table I that in order to obtain a scalar spectral index close to 0.96, the energy scale for observable inflation is typically on the order of 10^{16} GeV.

So far, we have been discussing the case in which $\phi < M$ during inflation. Alternatively, the inflaton may roll toward its minimum starting from values larger than the VEV, similar to simple models of chaotic inflation [9]. This is true in both of the inflationary models (CW and Higgs) that we consider here. For shorthand, we henceforth denote these regimes as the BV (below VEV) and AV (above VEV) branches. In contrast to the case of BV inflation described above, the AV case exhibits SU(5) breaking throughout inflation, and so the analysis of supercooling in this regime is even simpler.

A. Results for the CW model

The inflationary slow-roll parameters are defined as [15]

$$\epsilon = \frac{1}{2} \left(\frac{V'}{V} \right)^2, \quad \eta = \left(\frac{V''}{V} \right), \quad \xi^2 = \left(\frac{V'V'''}{V^2} \right). \quad (9)$$

(Here and below we use units $m_p = 1$, where $m_p \simeq 2.4 \times 10^{18}$ GeV is the reduced Planck mass.) The slow-roll approximation is valid as long as the conditions $\epsilon \ll 1$ and $\eta \ll 1$ hold. In this case the spectral index n_s , the tensor-to-scalar ratio r , and the running of the spectral index $\frac{dn_s}{d \ln k}$ are given by

$$n_s \simeq 1 - 6\epsilon + 2\eta, \quad (10)$$

$$r \simeq 16\epsilon, \quad (11)$$

$$\frac{dn_s}{d \ln k} \simeq 16\epsilon\eta - 24\epsilon^2 - 2\xi^2. \quad (12)$$

The number of e-foldings after the comoving scale $l_0 = 2\pi/k_0$ has crossed the horizon is given by

$$N_0 = \frac{1}{2} \int_{\phi_e}^{\phi_0} \frac{H(\phi) d\phi}{H'(\phi)}, \quad (13)$$

where ϕ_0 is the value of the field when the scale corresponding to k_0 exits the horizon, and ϕ_e is the value of the field at the end of inflation. The value of ϕ_e is given by the condition $2(H'(\phi_e)/H(\phi_e))^2 = 1$, which can be calculated from the Hamilton-Jacobi equation [16]

$$[H'(\phi)]^2 - \frac{3}{2}H^2(\phi) = -\frac{1}{2}V(\phi). \quad (14)$$

The amplitude of the curvature perturbation $\Delta_{\mathcal{R}}$ is given by

$$\Delta_{\mathcal{R}} = \frac{1}{2\sqrt{3}\pi} \frac{V^{3/2}}{|V'|}. \quad (15)$$

(Note that, for added precision, we include in our calculations the first order corrections in the slow-roll expansion for the quantities n_s , r , and $\Delta_{\mathcal{R}}$ [17].)

To calculate the magnitude of A and the inflationary parameters, we use these standard equations above. The WMAP5 value for $\Delta_{\mathcal{R}}$ is 4.91×10^{-5} for $k_0 = 0.002 \text{ Mpc}^{-1}$ [5]. The observable number of e-foldings corresponding to the same scale is

$$N_0 \simeq 53 + \frac{2}{3} \ln \left[\frac{V(\phi_0)^{1/4}}{10^{15} \text{ GeV}} \right] + \frac{1}{3} \ln \left[\frac{T_r}{10^9 \text{ GeV}} \right]. \quad (16)$$

For our calculations here and in the next section, we use the formula given in Eq. (28) to obtain values for the reheat temperature T_r . This formula is based on an SO(10) GUT model and yields values on the order of 10^6 – 10^8 GeV. In contrast, SU(5) considerations give $T_r \sim 10^9$ GeV.

Our predictions for the values of various parameters are displayed in Table I. As we see in Fig. 1, both the BV and AV branches fall within the WMAP5 2σ bounds for some range, and part of the BV branch lies inside the 1σ bound. As discussed earlier, larger values of $V_0^{1/4}$ result in observable inflation occurring closer to the VEV M . For the largest values of $V_0^{1/4}$, the potential is effectively given by $V = \frac{1}{2}m_\phi^2(\Delta\phi)^2$, where $\Delta\phi = M - \phi$ denotes the deviation of the field from the minimum and $m_\phi = 2\sqrt{AM} (= \frac{4\sqrt{V_0}}{M})$ is the inflaton mass in the CW model. This well-

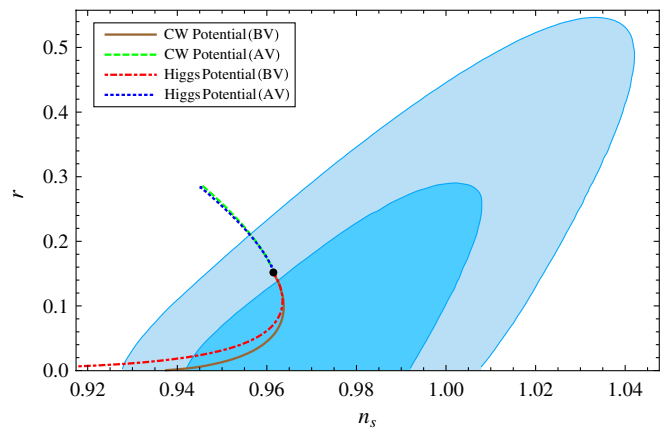


FIG. 1 (color online). r vs n_s for the Higgs and Coleman-Weinberg models, shown together with the WMAP5 contours (68% and 95% confidence levels) [5]. In each model, inflation is allowed both below the VEV (BV) and above the VEV (AV). The BV and AV branches for the Higgs (CW) potential are shown in the red, dot-dashed line (brown, solid line) and blue, dotted line (green, dashed line), respectively. The black circle corresponds to a quadratic potential with $N_0 = 52$, $n_s = 0.961$, and $r = 0.152$.

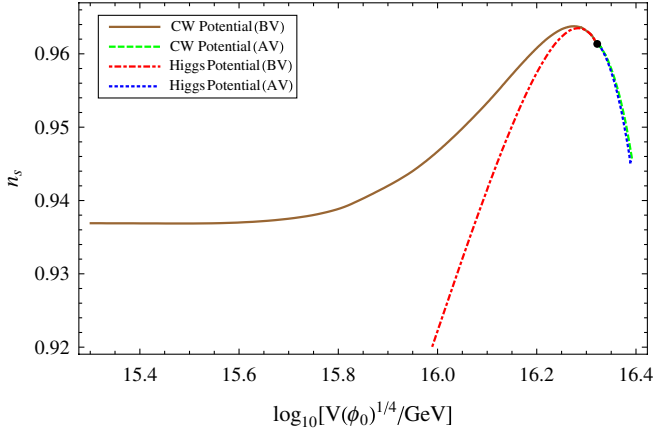


FIG. 2 (color online). n_s vs $\log_{10}[V(\phi_0)^{1/4}/\text{GeV}]$ for the CW and Higgs models. The BV and AV branches for the Higgs (CW) potential are shown in the red, dot-dashed line (brown, solid line) and blue, dotted line (green, dashed line), respectively. The black circle corresponds to a quadratic potential with $N_0 = 52$ and $n_s = 0.961$.

known monomial model [14] predicts $m_\phi \simeq 2 \times 10^{13}$ GeV, $\Delta\phi_0 \simeq 2\sqrt{N_0}$, $n_s \simeq 1 - \frac{2}{N_0}$, and $r \simeq 4(1 - n_s)$, corresponding to $V(\phi_0) \simeq (2 \times 10^{16} \text{ GeV})^4$. This is the region in which the two branches meet, i.e. both the BV and AV branches converge to quadratic inflation in the high- V_0 limit.

In the opposite limit with lower values of V_0 , the BV branch reduces to new inflation models with an effective potential $V = V_0(1 - |\lambda_\phi|(\frac{\phi}{M})^4)$, where $\lambda_\phi = \ln(\frac{\phi}{M})^4$, leading to $n_s \simeq 1 - \frac{3}{N_0}$ and $r \simeq \frac{16}{3}(1 - n_s)|\lambda_{\phi_0}|(\frac{\phi_0}{M})^4 \approx 0$. On the other hand, the AV branch is asymptotic to quartic

inflation with an effective potential $V = (\frac{V_0}{M^4})\lambda_\phi\phi^4$, resulting in $n_s \simeq 1 - \frac{3}{N_0}$ and $r \simeq \frac{16}{3}(1 - n_s)$.

As previously indicated, Table I shows that the energy scale of observable inflation in this model is $\sim 10^{16}$ GeV. It is instructive to see how n_s changes with this quantity, and this is displayed in Fig. 2.

III. GUT INFLATION WITH A HIGGS POTENTIAL

In this section we implement inflation by employing a Higgs potential given by [9,18,19]

$$V(\phi) = V_0 - \mu^2\phi^2 + \lambda\phi^4. \quad (17)$$

It is useful to rewrite this in terms of the vacuum potential V_0 and the vacuum expectation value M of the inflaton [9,19]:

$$V(\phi) = V_0 \left[1 - \left(\frac{\phi}{M} \right)^2 \right]^2. \quad (18)$$

As mentioned earlier, and previously discussed in Ref. [9], inflation may occur above or below the VEV M . We proceed in the same way as before, noting that V_0 is not related to other parameters in a simple way as it was in the

CW case. In this model, the inflaton mass is given by $m_\phi = \frac{2\sqrt{2V_0}}{M}$.

The results of this calculation are given in Table II. For small values of V_0 , the AV branch approaches quartic inflation with an effective potential $V = (\frac{V_0}{M^4})\phi^4$, which is the same as in the CW case apart from the slowly varying function λ_ϕ . On the other hand, the BV branch approaches new inflation with an effective potential $V = V_0(1 - 2(\frac{\phi}{M})^2)$ in a region disfavored by WMAP5. This leads to

TABLE II. Predicted values of various inflationary parameters using the Higgs potential given in Eq. (18), analogous to Table I.

$V_0^{1/4}$ (GeV)	$V(\phi_0)^{1/4}$ (GeV)	M	ϕ_0	ϕ_e	n_s	r	T_r (GeV)	$\frac{dn_s}{d \ln k} (-10^{-3})$
1.19×10^{16}	1.18×10^{16}	11.57	1.482	10.63	0.9360	0.0151	1.47×10^7	0.316
1.37×10^{16}	1.35×10^{16}	13.26	2.482	12.32	0.9466	0.0259	1.19×10^7	0.427
1.58×10^{16}	1.52×10^{16}	15.86	4.331	14.90	0.9550	0.0424	9.44×10^6	0.537
1.83×10^{16}	1.68×10^{16}	20.00	7.727	19.03	0.9604	0.0638	7.28×10^6	0.621
2.11×10^{16}	1.81×10^{16}	26.41	13.50	25.43	0.9628	0.0855	5.49×10^6	0.669
2.44×10^{16}	1.90×10^{16}	35.69	22.32	34.70	0.9635	0.103	4.09×10^6	0.692
2.89×10^{16}	1.97×10^{16}	51.57	37.83	50.58	0.9634	0.119	2.86×10^6	0.708
3.64×10^{16}	2.02×10^{16}	83.82	69.79	82.82	0.9629	0.131	1.78×10^6	0.719
4.33×10^{16}	2.04×10^{16}	119.9	105.7	118.9	0.9626	0.137	1.25×10^6	0.726
4.33×10^{16}	2.12×10^{16}	129.5	144.2	130.5	0.9611	0.161	1.21×10^6	0.739
3.64×10^{16}	2.13×10^{16}	93.31	108.1	94.33	0.9608	0.164	1.69×10^6	0.738
2.89×10^{16}	2.16×10^{16}	60.86	75.89	61.89	0.9603	0.171	2.63×10^6	0.739
2.44×10^{16}	2.18×10^{16}	44.60	59.82	45.63	0.9597	0.178	3.66×10^6	0.742
2.11×10^{16}	2.20×10^{16}	34.71	50.13	35.74	0.9591	0.185	4.79×10^6	0.745
1.83×10^{16}	2.22×10^{16}	27.24	42.88	28.28	0.9583	0.192	6.24×10^6	0.751
1.58×10^{16}	2.24×10^{16}	21.58	37.46	22.63	0.9573	0.201	8.09×10^6	0.760
1.37×10^{16}	2.27×10^{16}	17.27	33.42	18.33	0.9563	0.210	1.04×10^7	0.771
1.19×10^{16}	2.29×10^{16}	13.97	30.40	15.04	0.9551	0.219	1.34×10^7	0.784
1.03×10^{16}	2.32×10^{16}	11.40	28.12	12.48	0.9539	0.228	1.71×10^7	0.802

$n_s \simeq 1 - \frac{2}{N_0} \ln\left(\frac{M}{\sqrt{2}\phi_0}\right)$ and $r \simeq 8(1 - n_s)e^{-N_0(1-n_s)}$. In contrast to the AV regime, the CW and Higgs models produce distinct predictions in the BV regime, which is apparent from Fig. 1. For large V_0 , both branches converge to quadratic inflation and to the CW solution. As in the CW model, observable inflation in this region takes place near the VEV M . Hence, it is not surprising that the two models yield very similar results.

As shown in Fig. 1, the Higgs model yields results compatible with WMAP5 for a wide range of values. Analogous to the CW case, BV inflation appears to be somewhat more favorable than AV inflation, although both branches fall within 2σ of the central value for some range. In this model, the energy scale of observable inflation is again $\sim(1-2) \times 10^{16}$ GeV, except for regions far outside the WMAP bounds. This can be seen in Table II for the most favored range of (r, n_s) values, as well as in Fig. 2.

Note that radiative corrections to the inflationary potential, especially from Yukawa interactions, can modify the tree-level inflationary predictions in Table II. This has recently been discussed for ϕ^2 and ϕ^4 potentials in Ref. [20]. Additionally, when using the Higgs model of inflation in conjunction with an SU(5) GUT, some care must be taken to ensure that SU(5) breaks in the desired way. In order for SU(5) to break to $SU(3) \times SU(2) \times U(1)$ and subsequently to $SU(3) \times U(1)$, we must take $a > -\frac{7}{15}b$, $b > 0$, and $\gamma < 0$ in Eq. (1) [21]. While this requirement must also be satisfied in the case of CW inflation, we have already chosen a more restricted set of couplings earlier in our discussion.

IV. REALISTIC SU(5) AND SO(10) MODELS

It is well known that satisfactory gauge coupling unification is not achieved in the minimal nonsupersymmetric SU(5) model [22]. However, this situation can be improved by introducing additional matter fields at a low energy scale. For example, following Ref. [23], if we introduce vectorlike fermions $Q(3, 2, 1/6) + \bar{Q}(\bar{3}, 2, -1/6)$ and $D_c(\bar{3}, 1, 1/3) + \bar{D}_c(3, 1, -1/3)$ with masses ~ 200 GeV, the gauge couplings unify at a scale $M_X \sim 3.8 \times 10^{16}$ GeV, as shown in Fig. 3. Now using $V_0^{1/4} \sim M_X/2$ and referring to Table I, we obtain a corresponding value of the spectral index $n_s \sim 0.962$ which is well inside the 1σ bound of WMAP5 (see Fig. 1).

As mentioned earlier an inflationary scenario should provide, among other things, both a suitable cold dark matter candidate and an explanation of the observed baryon asymmetry. We will now consider a class of SO(10) GUT models in which these conditions are readily met [24]. In these models axions comprise the dark matter of the universe, and the observed baryon asymmetry arises via leptogenesis [25,26]. Clearly, axion dark matter can also be introduced in the SU(5) model.

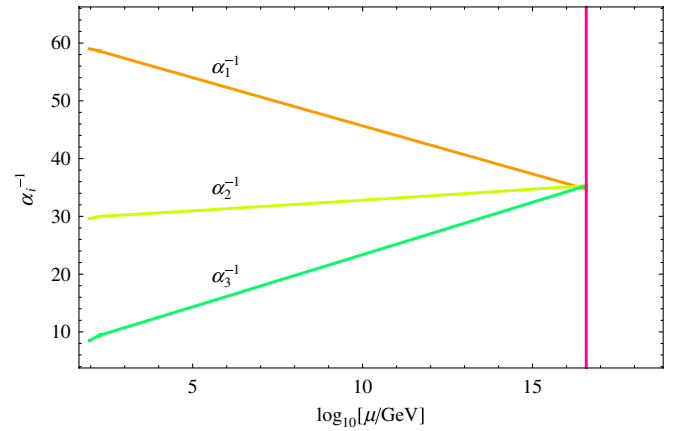


FIG. 3 (color online). Gauge coupling unification in the SU(5) model with additional fermions $Q + \bar{Q} + D_c + \bar{D}_c$ at mass scale ~ 200 GeV.

The breaking of SO(10) to the standard model (SM) proceeds via an intermediate step $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ [27]. The $U(1)_{PQ}$ (Peccei-Quinn) symmetry and $SU(2)_R \times U(1)_{B-L}$ are both spontaneously broken at some scale M_{B-L} . Axion dark matter physics requires that M_{B-L} is around $10^{11}-10^{12}$ GeV [28]. Furthermore, from our earlier discussion, satisfactory CW inflation requires that the SO(10) symmetry breaking scale be close to, indeed somewhat larger than 10^{16} GeV. It is intriguing that the scale of $B-L$ and axion symmetry breaking and the inflaton mass are of the same order of magnitude. This enables the inflaton to produce right-handed neutrinos whose subsequent decay produces the observed baryon asymmetry via nonthermal leptogenesis [26].

The fermion content of the SO(10) model consists of three SM families in the 16-dimensional spinor representations, as well as two fermion matter multiplets in the 10-plet representations [24]. These 10-plet fields are included in order to resolve the well-known axion domain-wall problem [29], by ensuring that a residual, discrete PQ symmetry coincides with the center, Z_4 , of SO(10) [30]. Under $U(1)_{PQ}$, the fermion fields transform as follows:

$$\begin{aligned} \psi_{16}^{(j)} &\rightarrow e^{i\theta} \psi_{16}^{(j)} (j = 1, 2, 3), \\ \psi_{10}^{(\alpha)} &\rightarrow e^{(-2i\theta)} \psi_{10}^{(\alpha)} (\alpha = 1, 2). \end{aligned} \quad (19)$$

The SO(10) symmetry breaking proceeds as follows:

$$\begin{aligned} \text{SO}(10) \times U(1)_{PQ} &\xrightarrow{210} \text{SU}(3)_c \times \text{SU}(2)_L \times \text{SU}(2)_R \\ &\quad \times U(1)_{B-L} \times U(1)_{PQ} \\ &\xrightarrow{45, 126} \text{SU}(3)_c \times \text{SU}(2)_L \times U(1)_Y \\ &\xrightarrow{10} \text{SU}(3)_c \times U(1)_{em}, \end{aligned} \quad (20)$$

where the Higgs fields necessary to implement this chain

are as indicated. Under $U(1)_{\text{PQ}}$, the Higgs fields transform as follows:

$$\begin{aligned} \phi^{(210)} &\rightarrow \phi^{(210)}, & \phi^{(126)} &\rightarrow e^{2i\theta} \phi^{(126)}, \\ \phi^{(45)} &\rightarrow e^{4i\theta} \phi^{(45)}, & \phi^{(10)} &\rightarrow e^{-2i\theta} \phi^{(10)}. \end{aligned} \quad (21)$$

As in the fermion case, these $U(1)_{\text{PQ}}$ transformation properties ensure that the action of the residual PQ symmetry on these fields is identical to that of the center of $\text{SO}(10)$. Note that all Higgs fields except for $\phi^{(210)}$ are complex. The allowed Yukawa couplings are (in schematic form)

$$\psi_{16} \psi_{16} \phi^{(10)}, \quad \psi_{16} \psi_{16} \phi^{(126)\dagger}, \quad \psi_{10} \psi_{10} \phi^{(45)}. \quad (22)$$

The allowed Higgs couplings include

$$\begin{aligned} \phi^{(210)} \phi^{(126)\dagger} \phi^{(126)\dagger} \phi^{(45)}, & \quad \phi^{(210)} \phi^{(126)\dagger} \phi^{(10)} \phi^{(45)}, \\ \phi^{(210)} \phi^{(126)} \phi^{(10)}. \end{aligned} \quad (23)$$

These couplings guarantee that $U(1)_{\text{PQ}}$ is the only global symmetry present. They also guarantee that $\phi^{(45)} \rightarrow -\phi^{(45)}$ is not a symmetry of the Lagrangian so that the domain-wall problems associated with this symmetry can be avoided [29]. Now, $\langle \phi^{(210)} \rangle$ cannot break $U(1)_{\text{PQ}}$ since it is neutral under $U(1)_{\text{PQ}}$. Hence $U(1)_{\text{PQ}}$ is broken at the intermediate scale M_{B-L} by $\langle \phi^{(126)} \rangle, \langle \phi^{(45)} \rangle$.

To make contact with the discussion on CW-based $\text{SU}(5)$ inflation, we need to make sure that the tree-level couplings in Eqs. (22) and (23) respect scale invariance. In particular, any cubic scalar coupling which respects $\text{SO}(10) \times U(1)_{\text{PQ}}$ is accompanied by the inflaton field ϕ and suitable dimensionless coefficients.

In its minimal form, the $\text{SO}(10)$ axion model includes the following Higgs contributions in the renormalization group equations (RGEs): the $(1, 1, 3, +1)$ and $(1, 3, 1, -1)$ components of the 126-plet, the $(1, 3, 1, 0)$ and $(1, 1, 3, 0)$ components of the 45-plet, and the $(1, 2, 2, 0)$ components of the 10-plet are included between M_X and M_{B-L} , while only the SM Higgs doublet in the $(1, 2, 2, 0)$ components of the 10-plet is included between M_{B-L} and M_Z . The fermions in the 10-plet acquire masses $\sim M_{B-L}$. Using two loop RGEs for the standard model gauge couplings [31], we find $M_{B-L} = 3.5 \times 10^{11}$ GeV and $M_X = 1.05 \times 10^{15}$ GeV.

To rectify the situation and obtain energy scales $\sim 10^{16}$ GeV, we consider threshold corrections from suitable (intermediate mass $\sim M_{B-L}$) scalar components in the 45 and 126 multiplets. To illustrate this, we consider two appropriate combinations of these scalar components. In order to simplify matters, we include only the left-right symmetric scalar components in each of these cases. For our first example, we take $(8, 1, 1, 0)$ from 45, $(3, 1, 1, \sqrt{3/2}(-1/6)) + (\bar{3}, 1, 1, \sqrt{3/2}(1/6))$ and $(3, 1, 3, \sqrt{3/2}(-1/6)) + (\bar{3}, 3, 1, \sqrt{3/2}(1/6))$ from 126 all at scale M_{B-L} . Employing the two loop RGEs for the gauge cou-

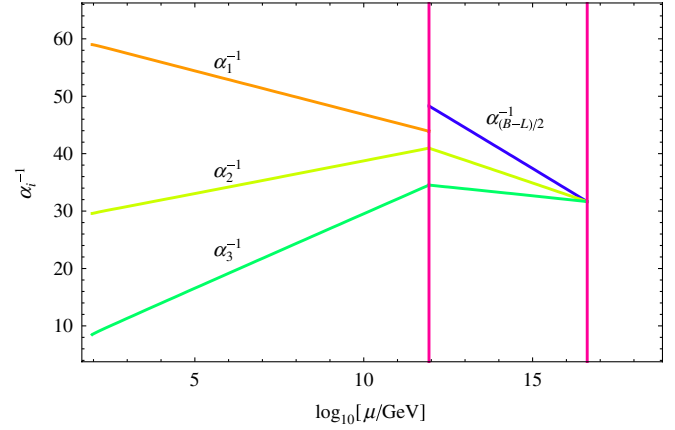


FIG. 4 (color online). Gauge coupling unification in the $\text{SO}(10)$ axion model at a scale $M_X = 4.13 \times 10^{16}$ GeV, with the addition of extra Higgs components at an intermediate scale $M_{B-L} = 8.54 \times 10^{11}$ GeV. Here $\alpha_1^{-1} = \frac{2}{5} \alpha_{(B-L)/2}^{-1} + \frac{3}{5} \alpha_2^{-1}$ at scale M_{B-L} .

plings, we obtain unification at $M_X = 2.61 \times 10^{16}$ GeV with $M_{B-L} = 8.0 \times 10^{11}$ GeV.

For our second example, we choose at scale M_{B-L} the following multiplets: $(8, 1, 1, 0)$ and $(3, 1, 1, \sqrt{3/2}(-2/6)) + (\bar{3}, 1, 1, \sqrt{3/2}(2/6))$ from 45, and $(1, 2, 2, 0)$, $(3, 1, 3, \sqrt{3/2}(-1/6)) + (\bar{3}, 3, 1, \sqrt{3/2}(1/6))$ and $(3, 2, 2, \sqrt{3/2}(-2/6)) + (\bar{3}, 2, 2, \sqrt{3/2}(2/6))$ from 126. In this case, we obtain unification at $M_X = 4.13 \times 10^{16}$ GeV with $M_{B-L} = 8.54 \times 10^{11}$ GeV (see Fig. 4).

V. REHEAT TEMPERATURE AND NONTHERMAL LEPTOGENESIS

In $\text{SU}(5)$, it is natural to consider the Yukawa coupling of the $\text{SU}(5)$ singlet inflaton field ϕ to right-handed neutrinos N_i ($i = 1, 2, 3$). For simplicity we consider a simple Yukawa coupling

$$Y_N \phi N N, \quad (24)$$

where $Y_N \sim M_N / \langle \phi \rangle$ is the coupling strength. For direct decay of the inflaton via this coupling, we take $2M_N \leq m_\phi$; then, using $\langle \phi \rangle \sim (10-100)m_P$, we expect $Y_N \leq 10^{-6}$. Thus, we estimate the maximum value of the reheat temperature T_r to be

$$T_r \sim \left(\frac{m_\phi}{M} \right) (m_\phi m_P)^{1/2} \sim 10^9 \text{ GeV}. \quad (25)$$

In the $\text{SO}(10)$ model, the value of T_r is estimated from the coupling [24]

$$126 \times \overline{126} \times \phi^2, \quad (26)$$

which carries a negative sign in front in order to induce the intermediate scale breaking. (There is a similar coupling involving the 45-plet.) The coefficient of the expression in Eq. (26) is of order $(M_{B-L} / \langle \phi \rangle)^2$. Similarly as above, we

take $2M_{B-L} \lesssim m_\phi$. The decay rate of the inflaton into the scalar components ($\overline{10}, 1, 3$) of the 126-plet is then given by

$$\Gamma_\phi \sim \left(\frac{M_{B-L}}{M}\right)^4 \frac{M^2}{m_\phi}. \quad (27)$$

The reheat temperature T_r is estimated to be

$$T_r \sim \sqrt{\Gamma_\phi m_P} \sim \left(\frac{M_{B-L}}{M}\right) \left(\sqrt{\frac{m_P}{m_\phi}}\right) M_{B-L}. \quad (28)$$

The scalar bosons produced by the inflaton rapidly decay through their Yukawa couplings into right-handed neutrinos. Under the assumption of hierarchical right-handed neutrino masses, the lepton asymmetry is given by [25,26,32]

$$n_L/s \lesssim 3 \times 10^{-10} \left(\frac{M_{N_i}}{M_{B-L}}\right) \left(\frac{T_r}{10^6 \text{ GeV}}\right) \left(\frac{m_{\nu_3}}{0.05 \text{ eV}}\right), \quad (29)$$

where M_{N_i} denotes the mass of the heaviest right-handed neutrino. From the experimental value of the baryon to photon ratio $\eta_B = 6.225 \times 10^{-10}$ [33], the required lepton asymmetry is found to be $n_L/s \approx 2.5 \times 10^{-10}$. With $T_r \sim (10^6-10^8)$ GeV, these heavy neutrinos, with masses on the order of $8 \times (10^9-10^{11})$ GeV or so, can give rise to the observed baryon asymmetry via nonthermal leptogenesis.

VI. MAGNETIC MONOPOLES, AXION DOMAIN WALLS, AND INFLATION

The spontaneous breaking of any GUT symmetry yields topologically stable magnetic monopoles. In our SU(5) and SO(10) examples, these monopoles carry masses of order $\frac{M_X}{\alpha_G}$, where $\alpha_G \sim \frac{1}{30} - \frac{1}{40}$ denotes the GUT coupling constant. In the SU(5) model, for instance, monopoles are produced during inflation once the magnitude of the coefficient $\lambda_2 \langle \phi \rangle^2$ associated with the $\text{Tr} \Sigma^2$ term in Eq. (1) exceeds $\sim H^2$, where $H = \sqrt{\frac{V_0}{3m_p^2}}$ ($\sim (0.01-4) \times 10^{14}$ GeV) denotes the Hubble constant during CW inflation. Using Table I and Eq. (3), it is readily checked that this occurs for

values of $\phi \ll \phi_0$. Thus, the corresponding number of e-foldings is much greater than 50–60 and the monopoles are inflated away. This holds for the inflaton field rolling to its minimum from values smaller than its present VEV M . Clearly, the same results hold if the inflaton rolls to its minimum from values that are larger than M .

The breaking of $U(1)_{\text{PQ}}$ symmetry occurs at intermediate scales $\sim 10^{12}$ GeV $\ll H$. In this case the corresponding phase transition is completed at the end of inflation, with $T_r \sim 10^6-10^9$ GeV. Thus, it is prudent to ensure that there are no stable domain walls in the model by including in the SO(10) model two 10-plets of new matter fields [see Eq. (19)].

VII. GUT INFLATION AND PROTON DECAY

Our analysis above based on GUT inflation shows that the vacuum energy during inflation, for both the CW and the Higgs potential, is of order 10^{16} GeV if we wish to obtain a scalar spectral index close to 0.96, as suggested by WMAP5. In the CW case, the SU(5) and SO(10) superheavy gauge bosons which mediate proton decay are then expected to possess a mass M_X which is a factor 2–4 larger than this scale. For the Higgs potential, the connection between the estimated scale of vacuum energy and M_X is somewhat tenuous, but it seems reasonable to assert that they are of comparable magnitude. Put differently, the value for M_X inferred above is a factor $\sim 1-10$ larger than the lower bound $\sim 4 \times 10^{15}$ GeV obtained from the observed stability of the proton [8]. Thus, we expect proton lifetime estimates based on gauge boson mediated decays to be more in line with what one finds in supersymmetric GUTs, where $M_X \sim 2 \times 10^{16}$ GeV [34]. A few lifetime estimates based on the naive expression for the decay rate [8]

$$\Gamma_p \approx \alpha_G^2 \frac{m_p^5}{M_X^4} \quad (30)$$

are presented in Table III. (Here m_p is the proton mass.)

TABLE III. Superheavy gauge bosons masses and corresponding proton lifetimes with $\alpha_G = \frac{1}{35}$ in the CW and Higgs models. Note that since the lifetime depends only on M_X , the results shown here apply equally well to the BV and AV branches in each model.

Coleman-Weinberg potential		Higgs potential	
$M_X \sim 2V_0^{1/4}$ (GeV)	$\tau(p \rightarrow \pi^0 e^+)$ (years)	$M_X \sim V_0^{1/4}$ (GeV)	$\tau(p \rightarrow \pi^0 e^+)$ (years)
5.0×10^{15}	1.8×10^{34}	1.0×10^{16}	2.8×10^{35}
1.0×10^{16}	2.8×10^{35}	1.2×10^{16}	5.8×10^{35}
1.2×10^{16}	5.8×10^{35}	1.4×10^{16}	1.1×10^{36}
1.8×10^{16}	2.9×10^{36}	1.6×10^{16}	1.8×10^{36}
2.2×10^{16}	6.6×10^{36}	1.8×10^{16}	2.9×10^{36}
2.7×10^{16}	1.5×10^{37}	2.1×10^{16}	5.5×10^{36}
3.5×10^{16}	4.2×10^{37}	2.4×10^{16}	9.3×10^{36}
6.0×10^{16}	3.6×10^{38}	2.9×10^{16}	2.0×10^{37}

The presence of an intermediate scale $\sim 10^{12}$ GeV in the SO(10) axion model leads to the appearance of intermediate mass scalar leptoquarks (3, 1, $-2/3$) which mediate proton decay. In this case, the proton decay rate is given by [8]

$$\Gamma_p \approx |Y_u Y_d|^2 \frac{m_p^5}{M_{B-L}^4}, \quad (31)$$

where $Y_{u,d}$ denote the Yukawa couplings of the u and d quarks, respectively. For an intermediate scale $\sim (0.5-1) \times 10^{12}$ GeV as in Fig. 4, the proton lifetime is estimated to be of order $10^{34}-10^{35}$ years.

VIII. SUMMARY

Assuming that inflation is associated with a GUT symmetry breaking phase transition (so that primordial GUT monopoles do not pose a cosmological problem), we have provided estimates for the energy scale of observable in-

flation which yield values for the scalar spectral index centered close to 0.96. For both CW and Higgs inflationary potentials, an estimate for the masses of the superheavy gauge bosons which mediate proton decay can then be derived. Masses of order 10^{16} GeV for these bosons are favored, yielding proton lifetimes of order $10^{34}-10^{38}$ years. In some SO(10) models with axion dark matter, lifetimes of order $10^{34}-10^{35}$ years are realized with proton decay being mediated by scalar leptoquarks possessing intermediate scale masses of order 10^{12} GeV.

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