

New Type of Seesaw Mechanism for Neutrino Masses

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In a wide class of unified models there is an additional (and possibly dominant) term in the neutrino mass formula that under the simplest assumption takes the form $M_\nu = (M_N + M_N^T)u/M_G$, where M_N is the neutrino Dirac mass matrix, and $u = O(M_W)$. This makes possible highly predictive models. A generalization of this form yields realistic neutrino masses and mixings more readily than the usual seesaw formula in some models. The conditions for resonant enhancement of leptogenesis can occur naturally in such models.

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Grand unified theories (GUTs) provide an elegant explanation of the magnitude of neutrino masses. In GUTs, the exchange of fields whose mass is of order $M_G \approx 2 \times 10^{16}$ GeV, the unification scale, can lead to light-neutrino masses of order the “seesaw” scale $v^2/M_G \approx 10^{-3}$ eV. [The parameter of $v = (\sqrt{2}G_F)^{-1/2} \approx 174$ GeV is the vacuum expectation value (VEV) which breaks the weak interaction gauge group $SU(2)_L \times U(1)_Y$.] The seesaw scale accords well with the order of magnitude of the neutrino masses inferred from atmospheric and solar neutrino oscillations ($\sqrt{\delta m_{\text{atm}}^2} \approx 5 \times 10^{-2}$ eV [1] and $\sqrt{\delta m_{\text{sol}}^2} \approx 8.5 \times 10^{-3}$ eV [2]). So far, two kinds of seesaw mechanisms have been widely discussed in the literature: the conventional or type I seesaw [3] and the triplet or type II seesaw [4]. These differ in the kinds of $O(M_G)$ fields whose exchange leads to light-neutrino masses. In this Letter we observe that a third kind of seesaw mechanism can operate in a wide class of unified theories based on $SO(10)$ or larger groups. We argue that this “type III” seesaw mechanism, as we call it, leads to a form of the light-neutrino mass matrix, M_ν , that may more readily fit the observed pattern of neutrino masses and mixings. The type III mechanism may also have advantages for leptogenesis. We will begin by reviewing type I and type II seesaw mechanisms.

In the type I or conventional seesaw, there are “right-handed” neutrinos, N_i^c , which have $O(M_G)$ Majorana masses with each other, described by the term $(M_R)_{ij}N_i^cN_j^c$, and $O(v)$ Dirac masses with the ordinary neutrinos, described by the term $(M_N)_{ij}\nu_iN_j^c = (Y_N)_{ij}\nu_iN_j^c\langle H_u \rangle$. (i, j are family indices.) Figure 1 shows how the exchange of the N_i^c leads to effective masses for the ν_i . These masses are given by the famous type I seesaw formula

$$M_\nu^I = -M_N M_R^{-1} M_N^T. \quad (1)$$

In the type II seesaw, there is an $SU(2)_L$ -triplet Higgs field, T , with mass m_T of order M_G that couples both to neutrinos $[(Y_T)_{ij}\nu_i\nu_jT]$ and to the $SU(2)_L \times U(1)_Y$ -breaking Higgs H_u . Integrating out the T leads to an

effective mass operator $(M_T)_{ij}\nu_i\nu_j$, with $M_T \propto Y_T\langle H_u \rangle^2/m_T \sim v^2/M_G$.

The type I and type II seesaw formulas can be understood as arising from block diagonalizing the complete mass matrix of the neutrinos and antineutrinos:

$$\mathcal{L}_{\nu \text{ mass}} = (\nu_i, N_i^c) \begin{pmatrix} (M_T)_{ij} & (M_N)_{ij} \\ (M_N^T)_{ij} & (M_R)_{ij} \end{pmatrix} \begin{pmatrix} \nu_j \\ N_j^c \end{pmatrix}, \quad (2)$$

with $M_R \sim M_G$, $M_N \sim v$, $M_T \sim v^2/M_G$, giving $M_\nu = M_\nu^I + M_\nu^{II} = -M_N M_R^{-1} M_N^T + M_T$, neglecting terms higher order in v/M_G .

In $SO(10)$ models, the ν_i and N_i^c both are contained in the spinor multiplet $\mathbf{16}_i$. Under the $SU(5)$ subgroup that contains the standard model group, the ν_i are in $\mathbf{5}$ and the N_i^c are in $\mathbf{1}$. Thus, we write, in an obvious notation, $\nu_i \subset \mathbf{5}(\mathbf{16}_i)$ and $N_i^c \subset \mathbf{1}(\mathbf{16}_i)$. The mass matrix M_R can arise in two simple ways in $SO(10)$, therefore. It can come from a renormalizable term of the form $(Y_R)_{ij}\mathbf{16}_i\mathbf{16}_j\mathbf{126}_H$ or from an effective nonrenormalizable term of the form

$$\mathcal{O}_R = (Y_R)_{ij}\mathbf{16}_i\mathbf{16}_j\overline{\mathbf{16}}_H\overline{\mathbf{16}}_H/M_G. \quad (3)$$

Both possibilities are much discussed in the literature. In the former case, the type II mechanism can operate, along with type I, since the Higgs multiplet $\mathbf{126}_H$ contains the triplet T . In the latter case, however, one expects $M_T = 0$ in Eq. (2), since the $\overline{\mathbf{16}}_H$ does not contain the triplet T . It is this latter case that can lead to the type III seesaw mechanism, as we shall now show.

The operator shown in Eq. (3), since it is a nonrenormalizable effective operator, must itself come from “integrating out” some heavy fields, as shown in the Fig. 2 diagram. We assume for simplicity that the fields integrated out, denoted S_m , are $SO(10)$ singlets, $\mathbf{1}_m$, though

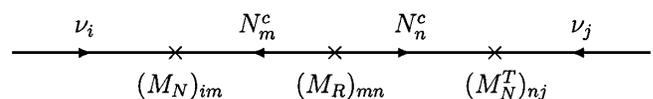


FIG. 1. Diagram that gives the light neutrinos type I seesaw masses of order v^2/M_G .

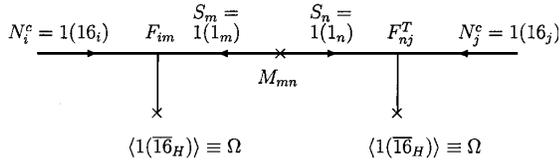


FIG. 2. Diagram that produces the effective operator $\mathbf{16}_i \mathbf{16}_j \overline{\mathbf{16}}_H \overline{\mathbf{16}}_H / M_G$, which generates M_R .

nothing in our later discussion depends on this. The couplings needed to produce the diagram in Fig. 2 are evidently $M_{mn} \mathbf{1}_m \mathbf{1}_n$ and $F_{im} \mathbf{16}_i \mathbf{1}_m \overline{\mathbf{16}}_H$. Altogether, this gives the well-known “double seesaw” mass matrix

$$\mathcal{L}_{\nu \text{ mass}} = (\nu_i, N_i^c, S_m) \begin{pmatrix} 0 & (M_N)_{ij} & 0 \\ (M_N^T)_{ij} & 0 & F_{in} \Omega \\ 0 & F_{mj}^T \Omega & M_{mn} \end{pmatrix} \begin{pmatrix} \nu_j \\ N_j^c \\ S_n \end{pmatrix}, \quad (4)$$

where $\Omega \equiv \langle \mathbf{1}(\overline{\mathbf{16}}_H) \rangle$, and where $i, j = 1, 2, 3$ and $m, n = 1, \dots, N$. N is the number of species of singlets S_m . It is easy to show that the effective mass matrix M_ν of the light neutrinos is given, up to negligible corrections higher order in ν/M_G , by

$$M_\nu = -M_N (F \Omega M^{-1} F^T \Omega)^{-1} M_N^T. \quad (5)$$

In other words, one has the usual type I seesaw formula with

$$M_R = (F \Omega) M^{-1} (F^T \Omega). \quad (6)$$

The name double seesaw refers to the fact that M_R arises from a seesaw involving N_i^c and S_m , and then produces masses for the light neutrinos via a seesaw involving ν_i and N_i^c .

So far, we have only taken into account the VEV of the SU(5)-singlet component of the $\overline{\mathbf{16}}_H$, which we called Ω . However, there is an SU(2)_L-doublet Higgs in the $\mathbf{5}(\overline{\mathbf{16}}_H)$ that can have an $O(\nu)$ VEV, which we shall call u . There is no *a priori* reason why u should vanish. If it does not, then the term $F_{im} \mathbf{16}_i \mathbf{1}_m \overline{\mathbf{16}}_H$ not only produces the $O(M_G)$ mass term $F_{im} (N_i^c S_m) \Omega$, but also an $O(\nu)$ mass term $F_{im} (\nu_i S_m) u$. Equation (4) then becomes

$$\mathcal{L}_{\nu \text{ mass}} = (\nu_i, N_i^c, S_m) \begin{pmatrix} 0 & (M_N)_{ij} & F_{in} u \\ (M_N^T)_{ij} & 0 & F_{in} \Omega \\ F_{mj}^T u & F_{mj}^T \Omega & M_{mn} \end{pmatrix} \begin{pmatrix} \nu_j \\ N_j^c \\ S_n \end{pmatrix}. \quad (7)$$

This can be simplified by a rotation in the $\nu_i N_i^c$ plane by angle $\tan^{-1}(u/\Omega)$: $\nu'_i = (\nu_i - \frac{u}{\Omega} N_i^c) / \sqrt{1 + (u/\Omega)^2}$, $N_i^{c'} = (N_i^c + \frac{u}{\Omega} \nu_i) / \sqrt{1 + (u/\Omega)^2}$, which has the effect of eliminating the νS entries. It also replaces the 0 in the $\nu\nu$ entry by

$$M_\nu^{\text{III}} = -(M_N + M_N^T) \frac{u}{\Omega}, \quad (8)$$

neglecting, as always, terms higher order in ν/M_G . This is the type III seesaw contribution. Otherwise, the resulting matrix has the same form as Eq. (4). Therefore, the full result for M_ν is given by the sum of Eqs. (5) and (8).

The relative size of the two contributions to M_ν is model dependent. Since M_N is related to the up quark mass matrix M_U by SO(10), one would expect the entries for the first and second families to be very small compared to ν . Consequently, because of the fact that M_N comes in squared in M_ν^{I} but only linearly in M_ν^{III} , the latter should dominate, except perhaps for the third family. M_ν^{III} would also dominate if the elements of M_{mn} were small compared to $\Omega \simeq M_G$, as Eqs. (5) and (6) show.

That M_ν^{III} dominates is an interesting possibility, as remarkably predictive SO(10) models of quark and lepton masses would then be constructable. Usually the most one can achieve in models where M_ν is given by the type I seesaw formula is predictions for the mass matrices of the up quarks, down quarks, and charged leptons (M_U, M_D, M_L), and for the Dirac mass matrix of the neutrinos (M_N), since these four matrices are intimately related to each other by symmetry. [For example, in the “minimal SO(10) model” they all come from one term $Y_{ij} \mathbf{16}_i \mathbf{16}_j \mathbf{10}_H$ and have exactly the same form.] However, sharp predictions for neutrino masses and mixings are hard to achieve because of the difficulty in constraining the form of M_R , which comes from different terms. On the other hand, if the type III seesaw contributions are dominant, then the matrix M_R is irrelevant; a knowledge of M_N and M_L is sufficient to determine the neutrino mass ratios and mixing angles.

In this Letter we are not so ambitious. Rather, we look at a version of the type III seesaw that is less predictive but still has certain attractive features. In the foregoing, we assumed that there was only a single $\overline{\mathbf{16}}$ of Higgs fields that contributed to neutrino masses. If there is more than one, then their coupling to neutrinos comes from the term $\sum_{aim} F_{im}^a (\mathbf{16}_i \mathbf{1}_m) \overline{\mathbf{16}}_{Ha}$, which contains $\sum_{im} F_{im} (N_i^c S_m) \Omega + \sum_{im} F'_{im} (\nu_i S_m) u$, where $F_{im} \equiv \sum_a F_{im}^a \Omega_a / \Omega$, $F'_{im} \equiv \sum_a F_{im}^a u_a / u$, $\Omega \equiv (\sum_a \Omega_a^2)^{1/2}$, and $u \equiv (\sum_a u_a^2)^{1/2}$. Then Eq. (7) is modified by having Yukawa matrices in the νS and $N^c S$ blocks that are no longer proportional to each other. It is then not possible to null out the νS block of M_ν by a simple flavor-independent rotation by angle $\tan^{-1}(u/\Omega)$, as in the special case discussed above. Consequently, the effective light-neutrino mass matrix is more complicated. In the most general case it can be written $M_\nu = -\tilde{M}_N M_R^{-1} \tilde{M}_N^T + (F' u) M^{-1} (F'^T u)$, where M_R is given by Eq. (6) as before, and $\tilde{M}_N \equiv M_N + (F' u) M^{-1} (F'^T \Omega)$. However, a great simplification results if one assumes that the number of species of singlet fermions S_m is three, i.e., one for each family. (If there were less than three, not all the N_i^c would get superlarge mass, and some of the light neutrinos would have masses of order ν .) With three species of S_m , the matrices F and F' are (generally) invertible and

then $M_\nu = M_\nu^I + M_\nu^{\text{III}}$, where M_ν^I is given as before by Eq. (5) and

$$M_\nu^{\text{III}} = -(M_N H + H^T M_N^T) \frac{u}{\Omega}, \quad H \equiv (F' F^{-1})^T. \quad (9)$$

In this the generalized type III seesaw formula, the dimensionless 3×3 matrix H introduces many unknown parameters, more, indeed, than does M_R in the type I seesaw. However, as we shall now show by an example, in SO(10) models it may be easier to obtain a realistic pattern of neutrino masses and mixings without fine-tuning the parameters in the generalized type III seesaw than in the type I seesaw.

The SO(10) model of Ref. [5] gives an excellent fit to the quark masses and mixings and the charged lepton masses, fitting 13 real quantities with eight real parameters. This fit uniquely determines the neutrino Dirac mass matrix (at the unification scale) to be

$$M_\nu = \begin{pmatrix} a_{11} \eta^2 & a_{13} \epsilon \eta & (a_{13} - a_{12} \epsilon) \eta \\ a_{13} \epsilon \eta & a_{33} \epsilon^2 & (a_{33} - a_{23} \epsilon) \epsilon \\ (a_{13} - a_{12} \epsilon) \eta & (a_{33} - a_{23} \epsilon) \epsilon & a_{33} - 2a_{23} \epsilon + a_{22} \epsilon^2 \end{pmatrix} m_U^2 / m_R. \quad (11)$$

Neglecting the relatively small first row and column, the condition that the ratio m_2/m_3 of the two heaviest neutrino masses be equal to some value r is that

$$a_{22} a_{33} - a_{23}^2 \cong \frac{r}{(1+r^2)^2} \left(a_{33} \frac{1}{\epsilon^2} - 2a_{23} \frac{1}{\epsilon} + a_{22} + a_{33} \right)^2. \quad (12)$$

It is evident that r naturally is of order $\epsilon^4 \approx 4 \times 10^{-4}$. For r to be of order ϵ^0 (as indicated by experiment, which gives $r \approx 1/6$) the elements must be somewhat ‘‘tuned.’’ For example, setting $a_{23}/a_{33} = p\epsilon^{-1} + O(\epsilon^0)$ and $a_{22}/a_{33} = q\epsilon^{-2} + O(\epsilon^{-1})$, Eq. (12) gives the condition $1 + 2p + q = 0$. In other words, not only must the 23 block of M_R have a hierarchy that is correlated with the hierarchy of the 23 block of M_N , but it must also satisfy a nontrivial numerical relation among its elements. This kind of mild fine-tuning of the 23 block of M_R with the

$$M_\nu = \begin{pmatrix} 0 & \epsilon H_{31} & H_{31} - \epsilon H_{21} \\ \epsilon H_{31} & 2\epsilon H_{32} & H_{32} + \epsilon(H_{33} - H_{22}) \\ H_{31} - \epsilon H_{21} & H_{32} + \epsilon(H_{33} - H_{22}) & 2(H_{33} - \epsilon H_{23}) \end{pmatrix} \frac{m_U u}{\Omega}. \quad (13)$$

Here it is clear that without any fine-tuning $|r|$ ($\equiv |m_2/m_3|$) is somewhat less than one, as desired. More precisely, $-r/(1+r^2) \cong \frac{1}{4}(H_{32}/H_{33})^2 + O(\epsilon)$. Moreover, the LMA solution naturally emerges. For U_{e3} to be consistent with present limits, ϵH_{21} must approximately cancel H_{31} in the 13 and 31 elements of M_ν . However, all the other elements of H can be of order one.

Note that a satisfactory pattern of light-neutrino masses and mixings emerges with *no hierarchy* among the superheavy neutrinos, which all have masses of order

$$M_N = \begin{pmatrix} \eta & 0 & 0 \\ 0 & 0 & \epsilon \\ 0 & -\epsilon & 1 \end{pmatrix} m_U, \quad (10)$$

where $m_U \cong m_t$, $\eta \cong m_u^0/m_t^0 \cong 0.6 \times 10^{-5}$, and $\epsilon \cong 3\sqrt{m_c^0/m_t^0} \cong 0.14$. (Superscripts here refer to quantities evaluated at M_G .) In this model there is a very large (namely, $\tan^{-1} 1.8$) contribution to the atmospheric neutrino mixing angle coming from the charged lepton mass matrix M_L , which is completely known. However, as M_R is not known, it is impossible to predict the neutrino mass ratios and the other neutrino mixing angles (or even the atmospheric angle precisely) within the framework of the type I seesaw. Nevertheless, one can ask whether what we know about these neutrino masses and mixings can be accommodated in the model with a reasonable form for M_R . Parametrizing that matrix by $(M_R^{-1})_{ij} = a_{ij} m_R^{-1} = a_{ji} m_R^{-1}$, the type I seesaw formula gives

addition of just four new parameters was used in [6] to obtain a very good fit to the large mixing angle (LMA) solution for the SO(10) model of [5]. Such fine-tuning is typically required in SO(10) models relying on the type I seesaw mechanism [7].

It can be seen from Eq. (11) that to fit the LMA solar solution $a_{11} \leq \epsilon^2/\eta^2$, $a_{12} \leq \epsilon/\eta$, and $a_{13} \sim \epsilon^2/\eta$. Thus, the correlation between the hierarchies of M_R and M_N extends also to the first family.

By contrast, a satisfactory pattern of neutrino masses and mixings can be achieved without any fine-tuning in this model if the type III seesaw mechanism dominates. There are two interesting cases. Suppose, first, that all the elements of F and of F' are of order f , a dimensionless parameter of order or smaller than 1. Then all the elements of $H \equiv (F' F^{-1})^T$ will be of order one. From Eq. (9), neglecting terms of order η ,

M_G , something that is impossible in the type I seesaw. This is an attractive possibility, but would create problems for thermal leptogenesis [8].

A second interesting case is that F and F' both have the form

$$F, F' \sim \begin{pmatrix} (\eta/\epsilon)^2 & \eta/\epsilon & \eta/\epsilon \\ \eta/\epsilon & 1 & 1 \\ \eta/\epsilon & 1 & 1 \end{pmatrix}, \quad (14)$$

as might arise naturally if the first family of both $\mathbf{16}_i$ and $\mathbf{1}_i$ had a different Abelian family charge than the other families. Then H has the form

$$H \sim \begin{pmatrix} 1 & \epsilon/\eta & \epsilon/\eta \\ \eta/\epsilon & 1 & 1 \\ \eta/\epsilon & 1 & 1 \end{pmatrix}. \quad (15)$$

Therefore, by Eq. (9), M_ν has the form

$$M_\nu \sim \begin{pmatrix} \eta & \epsilon & \epsilon \\ \epsilon & \epsilon & 1 \\ \epsilon & 1 & 1 \end{pmatrix} \frac{m_U u}{\Omega}, \quad (16)$$

that is, the same form as the previous case, except that U_{e3} is automatically of order ϵ .

The superheavy neutrinos do not consist here of three Majorana fermions, as in the standard type I seesaw mechanism, but of six Majorana fermions that combine (if M_{ij} is small compared to $F_{ij}\Omega$, as we are assuming) to form three pseudo-Dirac pairs. In the basis where F_{ij} is diagonal, there are the mass terms $M_1 N_1^c S_1 + M_2 N_2^c S_2 + M_3 N_3^c S_3 + \sum_{ij} M_{ij} S_i S_j$, where $M_1 \equiv F_{11}\Omega \sim (\eta/\epsilon)^2 M_G \sim 10^8$ GeV, and $M_2 \equiv F_{22}\Omega$ and $M_3 \equiv F_{33}\Omega$ are of order $M_G \sim 10^{16}$ GeV. The lightest of these states are of sufficiently small mass to allow thermal leptogenesis with a reheating temperature that is low enough to avoid the cosmological gravitino problem.

The type III seesaw has a feature that is advantageous for leptogenesis in certain types of models. It is often necessary in order to get sufficient leptogenesis in realistic SO(10) models for there to be a resonant enhancement [9] caused by the two lightest “right-handed” neutrinos forming a pseudo-Dirac pair (i.e., equivalently, a pair of Majorana fermions with nearly equal and opposite mass). Having such a pseudo-Dirac pair in the standard type I seesaw imposes a nontrivial constraint on the form of the matrix M_R . This constraint can clash with what is required in order to get a realistic M_ν through the type I seesaw formula. Indeed, this is the case in the realistic fermion mass model of Ref. [5] that we have been using as an illustration: a severe fine-tuning of M_R is required to have both satisfactory leptogenesis and realistic M_ν , as shown in Ref. [10]. In the type III seesaw, on the other hand, there are pseudo-Dirac pairs of neutrinos automatically present. And, under the assumption stated above about the smallness of M_{ij} , the pseudo-Dirac pair (N_1^c, S_1) is slightly split, so that it is equivalent to two

Majorana neutrinos with masses approximately given by $M_1 + \frac{1}{2}M_{11}$ and $-M_1 + \frac{1}{2}M_{11}$. No special constraint on the forms of F_{ij} , F'_{ij} , or H_{ij} is required to have this near degeneracy condition satisfied except that M_{ij} be small compared to $F_{ij}\Omega$ in the original basis (a condition that also makes the type III contribution to M_ν dominant). Hence, no clash between the requirements of leptogenesis and of realistic M_ν occurs. Indeed, in Ref. [10] it is shown that in the realistic model of Ref. [5], which we have been using as an example, sufficient leptogenesis can be obtained without any fine-tuning in the type III seesaw.

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