

Noncanonical MSSM, unification, and new particles at the CERN LHCIlia Gogoladze,^{1,*} Tianjun Li,^{2,3,†} V. N. Şenoğuz,^{4,‡} and Qaisar Shafi^{4,§}¹*Department of Physics and Astronomy, University of Delaware, Newark, Delaware 19716, USA*²*Department of Physics and Astronomy, Rutgers University, Piscataway, New Jersey 08854, USA*³*Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100080, People's Republic of China*⁴*Bartol Research Institute, Department of Physics and Astronomy, University of Delaware, Newark, Delaware 19716, USA*

(Received 25 August 2006; published 15 December 2006)

We consider noncanonical embeddings of the MSSM in high-dimensional orbifold grand unified theories (GUTs) based on the gauge symmetry $SU(N)$, $N = 5, 6, 7, 8$. The hypercharge normalization factor k_Y can either have unique noncanonical values, such as $23/21$ in a six-dimensional $SU(7)$ model, or may lie in a (continuous) interval. Gauge coupling unification and gauge-Yukawa unification can be realized in these models by introducing new particles with masses in the TeV range which may be found at the LHC. In one such example there exist color singlet fractionally charged states.

DOI: [10.1103/PhysRevD.74.126006](https://doi.org/10.1103/PhysRevD.74.126006)

PACS numbers: 11.25.Mj, 12.10.Kt

I. INTRODUCTION

High-dimensional orbifold grand unified theories (GUTs) [1,2] provide the elegant solutions to the well-known problems encountered in four-dimensional (4D) GUTs such as $SU(5)$ and $SO(10)$, especially the doublet-triplet splitting problem and the proton decay problem. The nonsupersymmetric version has, in particular, been exploited to show that unification of the standard model (SM) gauge couplings can be realized with a noncanonical embedding of $U(1)_Y$, the hypercharge component of the SM gauge group [3]. The couplings unify at $M_{\text{GUT}} \simeq 4 \times 10^{16}$ GeV, which is also the scale at which the 4D $\mathcal{N} = 1$ supersymmetry (SUSY) is broken, without introducing additional new particles. This approach has been taken a step further along two different directions. In [4] it was shown that by implementing additional gauge-Yukawa unification, the SM Higgs mass can be predicted. The mass turns out to be $135 \pm 6(144 \pm 4)$ GeV with gauge-top (bottom/tau) Yukawa unification. This is encouraging because it is different from the prediction of $\simeq 130$ GeV in the minimal supersymmetric standard model (MSSM). In [5] these ideas were extended to the case of split supersymmetry, with similar predictions for the Higgs mass.

The orbifold scenario for the GUT breakings assume the supersymmetric GUT models exist in high dimensions and are broken to 4D $\mathcal{N} = 1$ supersymmetric standard-like models for the zero modes due to the discrete symmetries on the extra space manifolds [1,2]. The zero modes can be identified with the low-energy SM fermions and Higgs fields, allowing gauge-Higgs unification [6] and gauge-Yukawa unification [7]. For the canonical $U(1)_Y$ normalization, the unification of the gauge couplings, top and

bottom quark Yukawa couplings, and τ lepton Yukawa coupling can be realized in the 6D orbifold $SU(8)$ and $SU(9)$ models, and cannot be obtained in the orbifold $SU(N)$ models with $N < 8$. Therefore, it is interesting to construct the minimal orbifold $SU(N)$ model with gauge-Yukawa unification.

In this paper, we show that the minimal model with the unification of the gauge couplings and third-family Yukawa couplings is the 6D orbifold $SU(7)$ model with noncanonical $U(1)_Y$ normalization $k_Y = 23/21$ where k_Y is defined in Eqs. (1) and (2). Moreover, we construct the 7D $SU(8)$ models with gauge-Yukawa unification and $k_Y > 23/21$. And for completeness, we consider the 6D orbifold $SU(5)$ and $SU(6)$ models with gauge-fermion and gauge-fermion-Higgs unification first as warm-up exercise.

The 4D gauge group in these models is $SU(3)_C \times SU(2)_L \times U(1)_Y$ accompanied by one or several extra $U(1)$ factors assumed to be broken at M_{GUT} . We define the unified gauge couplings at the GUT scale (M_{GUT}) as

$$g_1^2 = g_2^2 = g_3^2, \quad (1)$$

where

$$g_1^2 \equiv k_Y g_Y^2, \quad (2)$$

where k_Y is the $U(1)_Y$ normalization factor, and the g_Y , g_2 , and g_3 are the gauge couplings for $U(1)_Y$, $SU(2)_L$, and $SU(3)_C$ gauge groups, respectively. For the canonical $U(1)_Y$ normalization, we have $k_Y = 5/3$.

For orbifold GUTs where all of the SM fermions and Higgs fields are placed on a 3-brane at an orbifold fixed point, we can have any positive normalization for $U(1)_Y$, i.e., k_Y is an arbitrary positive real number. However, in this case charge quantization cannot be realized. We wish to consider the more interesting orbifold GUTs in which at least one of the SM fermions and Higgs fields arise from the zero modes of the bulk vector multiplet and their $U(1)_Y$ charges can be determined. The charge quantization can be

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achieved due to the gauge invariance of Yukawa couplings and anomaly free conditions. In the orbifold models we consider, k_Y is then either uniquely determined to have a noncanonical value or lies in a continuous interval. For the latter case $k_Y = 5/3$ is possible, but there is no apparent reason why this value would be realized.

Since the three SM gauge couplings unify quite nicely with the canonical hypercharge normalization, it can be argued that we should simply discard the models which do not predict $k_Y = 5/3$. However, unification in MSSM with $k_Y = 5/3$ may well be accidental, and as the example of nonsupersymmetric unification shows there are different possibilities. In this paper we assume a noncanonical hypercharge normalization as the models under consideration generally predict. We then discuss how gauge coupling unification and gauge-Yukawa unification can be obtained by adding a minimal set of vectorlike particles to the MSSM spectrum. It is certainly our hope that these vectorlike particles will be found at the Large Hadron Collider (LHC).

The paper is organized as follows. In Secs. II and III we consider $SU(5)$ and $SU(6)$ models. In the $SU(5)$ model the only zero mode that can be introduced in the bulk is a quark doublet and k_Y is predicted to be $1/15$. The model can be extended to $SU(6)$, with $k_Y \geq 1/15$. We construct two $SU(6)$ models with gauge-top and gauge-bottom Yukawa coupling unification, with $k_Y = 2/3$ and $1/3$, respectively. We discuss $SU(7)$ and $SU(8)$ models in Secs. IV and V. We can have gauge-Yukawa unification for the third family in an $SU(7)$ model, with $k_Y = 23/21$. This model can be extended to $SU(8)$, with $k_Y \geq 23/21$. Sections VI and VII concern gauge coupling unification and gauge-Yukawa unification with new particles in these models. We briefly remark on the Higgs mass in Sec. VIII and conclude in Sec. IX. Some details of the 6D and 7D orbifold models are provided in the two appendices.

II. $SU(5)$ MODELS

We consider a 6D $\mathcal{N} = (1, 1)$ supersymmetric $SU(5)$ gauge theory compactified on the orbifold $M^4 \times T^2/Z_6$ (for some details see Appendix A). The $\mathcal{N} = (1, 1)$ supersymmetry in 6D has 16 supercharges and corresponds to $\mathcal{N} = 4$ supersymmetry in 4D, and thus only the gauge multiplet can be introduced in the bulk. This multiplet can be decomposed under the 4D $\mathcal{N} = 1$ supersymmetry into a vector multiplet V and three chiral multiplets Σ_1, Σ_2 , and Σ_3 in the adjoint representation, where the fifth and sixth components of the gauge field, A_5 and A_6 , are contained in the lowest component of Σ_1 .

To break the $SU(5)$ gauge symmetry, we choose the following 5×5 matrix representation for R_{Γ_T} ,

$$R_{\Gamma_T} = \text{diag}(+1, +1, +1, \omega^{n_1}, \omega^{n_1}), \quad (3)$$

where $w = e^{i\pi/3}$ and $n_1 = 0$. Then, we obtain¹

$$SU(5)/R_{\Gamma_T} = SU(3)_C \times SU(2)_L \times U(1)_Y. \quad (5)$$

So, for the zero modes, the 6D $\mathcal{N} = (1, 1)$ supersymmetric $SU(5)$ gauge symmetry is broken down to 4D $\mathcal{N} = 1$ supersymmetric $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetry [2].

We define the generator for $U(1)_Y$ as follows:

$$T_{U(1)_Y} \equiv \frac{1}{30} \text{diag}(2, 2, 2, -3, -3). \quad (6)$$

Because $\text{tr}[T_{U(1)_Y}^2] = 1/30$, we obtain $k_Y = 1/15$.

Under $SU(3)_C \times SU(2)_L \times U(1)_Y$, the adjoint representation $\mathbf{24}$ of $SU(5)$ decomposes as

$$\mathbf{24} = \left(\begin{array}{cc} (\mathbf{8}, \mathbf{1})_{Q00} & (\mathbf{3}, \bar{\mathbf{2}})_{Q12} \\ (\bar{\mathbf{3}}, \mathbf{2})_{Q21} & (\mathbf{1}, \mathbf{3})_{Q00} \end{array} \right) + (\mathbf{1}, \mathbf{1})_{Q00}, \quad (7)$$

where the last term $(\mathbf{1}, \mathbf{1})_{Q00}$ denotes the gauge field associated with $U(1)_Y$. The subscripts Qij , with $Qij = -Qji$, denote the charges under $U(1)_Y$, and

$$Q00 = \mathbf{0}, \quad Q12 = \frac{1}{6}. \quad (8)$$

The Z_6 transformation properties for the decomposed components of V, Σ_1, Σ_2 , and Σ_3 are given by the first 2×2 submatrices in Eqs. (A9)–(A12) in Appendix A. We choose

$$k = 1, \quad n_1 = 5, \quad (9)$$

where k is given in Eqs. (A7) and (A8) in Appendix A. There are no zero modes from the chiral multiplets Σ_2 and Σ_3 , and only one zero mode, $(\mathbf{3}, \bar{\mathbf{2}})_{Q12}$, from the chiral multiplet Σ_1 , which can be identified as the third-family quark doublet Q_3 . The remaining MSSM matter fields and the two MSSM Higgs doublets can be put on the 3-brane at $z = 0$, where only the SM gauge symmetry is preserved.

III. $SU(6)$ MODELS

For the $SU(6)$ models where at least one of the SM fermions and Higgs fields arise from the zero modes of the chiral multiplets Σ_1, Σ_2 , and Σ_3 , we can show that the minimal normalization k_Y for $U(1)_Y$ is $1/15$, and the corresponding zero mode is quark doublet because it has the smallest $U(1)_Y$ quantum number. Moreover, we can only have the gauge-top or gauge-bottom quark Yukawa coupling unification, and we cannot obtain the right-handed leptons from the zero modes of the bulk vector multiplet.

In the following subsections, we present three $SU(6)$ models. In the first, the third-family quark doublet Q_3 is the

¹Suppose G is a Lie group and H is a subgroup of G , we denote the commutant of H in G as G/H , i.e.,

$$G/H \equiv \{g \in G | gh = hg, \text{ for any } h \in H\}. \quad (4)$$

only zero mode from the bulk vector multiplet, and k_Y is an arbitrary positive real number that is larger than or equal to $1/15$. In the second and third $SU(6)$ models, we have gauge-top and gauge-bottom quark Yukawa coupling unification, respectively. We consider 7D $\mathcal{N} = 1$ supersymmetric $SU(6)$ compactified on the orbifold $M^4 \times T^2/Z_6 \times S^1/Z_2$ (for some details see Appendix B), and 6D $\mathcal{N} = (1, 1)$ supersymmetric $SU(6)$ compactified on the orbifold $M^4 \times T^2/Z_6$. The compactification process yields 4D $\mathcal{N} = 1$ supersymmetric $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_\alpha$.

The generators for $U(1)_Y \times U(1)_\alpha$ are defined as follows:

$$\begin{aligned} T_{U(1)_Y} &\equiv \frac{1}{30} \text{diag}(2, 2, 2, -3, -3, 0) \\ &\quad + a \text{diag}(1, 1, 1, 1, 1, -5), \\ T_{U(1)_\alpha} &\equiv \text{diag}(2, 2, 2, -3, -3, 0) \\ &\quad - \frac{1}{30a} \text{diag}(1, 1, 1, 1, 1, -5), \end{aligned} \quad (10)$$

where a is a real number. Because $\text{tr}[T_{U(1)_Y}^2] = 1/30 + 30a^2$, we obtain

$$k_Y = \frac{1}{15} + 60a^2 \geq \frac{1}{15}. \quad (11)$$

The adjoint representation **35** of $SU(6)$ is decomposed under $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_\alpha$ as

$$\mathbf{35} = \begin{pmatrix} (\mathbf{8}, \mathbf{1})_{Q00} & (\mathbf{3}, \bar{\mathbf{2}})_{Q12} & (\mathbf{3}, \mathbf{1})_{Q13} \\ (\bar{\mathbf{3}}, \mathbf{2})_{Q21} & (\mathbf{1}, \mathbf{3})_{Q00} & (\mathbf{1}, \mathbf{2})_{Q23} \\ (\bar{\mathbf{3}}, \mathbf{1})_{Q31} & (\mathbf{1}, \bar{\mathbf{2}})_{Q32} & (\mathbf{1}, \mathbf{1})_{Q00} \end{pmatrix} + (\mathbf{1}, \mathbf{1})_{Q00}, \quad (12)$$

where $(\mathbf{1}, \mathbf{1})_{Q00}$ in the third diagonal entry of the matrix and the last term $(\mathbf{1}, \mathbf{1})_{Q00}$ denote gauge fields associated with $U(1)_Y \times U(1)_\alpha$. The subscripts Qij , with $Qij = -Qji$, are the charges under $U(1)_Y \times U(1)_\alpha$. The subscript $Q00 = (\mathbf{0}, \mathbf{0})$, and the other subscripts Qij with $i \neq j$ are

$$\begin{aligned} Q12 &= \left(\frac{\mathbf{1}}{\mathbf{6}}, \mathbf{5}\right), & Q13 &= \left(\frac{\mathbf{1}}{\mathbf{15}} + \mathbf{6a}, \mathbf{2} - \frac{\mathbf{1}}{\mathbf{5a}}\right), \\ Q23 &= \left(-\frac{\mathbf{1}}{\mathbf{10}} + \mathbf{6a}, -\mathbf{3} - \frac{\mathbf{1}}{\mathbf{5a}}\right). \end{aligned} \quad (13)$$

We will consider the following three models.

A. $SU(6)$ model I

Here the third-family quark doublet Q_3 is the only zero mode from the bulk vector multiplet, a is an arbitrary real number, and we have

$$k_Y \geq \frac{1}{15}. \quad (14)$$

To project out all the unwanted components in the chiral multiplets, we consider the 7D $\mathcal{N} = 1$ supersymmetric $SU(6)$. The $\mathcal{N} = 1$ supersymmetry in 7D has 16 super-

charges corresponding to $\mathcal{N} = 4$ supersymmetry in 4D, and only the gauge supermultiplet can be introduced in the bulk. This multiplet can be decomposed under 4D $\mathcal{N} = 1$ supersymmetry into a gauge vector multiplet V and three chiral multiplets Σ_1 , Σ_2 , and Σ_3 all in the adjoint representation, where the fifth and sixth components of the gauge field, A_5 and A_6 , are contained in the lowest component of Σ_1 , and the seventh component of the gauge field A_7 is contained in the lowest component of Σ_2 .

To break the $SU(6)$ gauge symmetry, we choose the following 6×6 matrix representations for R_{Γ_T} and R_{Γ_S}

$$R_{\Gamma_T} = \text{diag}(+1, +1, +1, \omega^{n_1}, \omega^{n_1}, \omega^{n_2}), \quad (15)$$

$$R_{\Gamma_S} = \text{diag}(+1, +1, +1, +1, +1, +1), \quad (16)$$

where $n_1 \neq n_2 \neq 0$.

Then, we obtain

$$SU(6)/R_{\Gamma_T} = SU(3) \times SU(2) \times U(1)_Y \times U(1)_\alpha, \quad (17)$$

$$SU(6)/R_{\Gamma_S} = SU(6), \quad (18)$$

$$SU(6)/\{R_{\Gamma_T} \cup R_{\Gamma_S}\} = SU(3) \times SU(2) \times U(1)_Y \times U(1)_\alpha. \quad (19)$$

Note that R_{Γ_S} only breaks the additional supersymmetry.

The $Z_6 \times Z_2$ transformation properties for the decomposed components of V , Σ_1 , Σ_2 , and Σ_3 are the 3×3 submatrices in Eqs. (B12)–(B15) in Appendix B where the third and fourth rows and columns are crossed out. We choose

$$n_1 = 5, \quad n_2 = 2, \quad \text{or} \quad 3. \quad (20)$$

Then, we obtain that there is no zero mode from the chiral multiplets Σ_2 and Σ_3 , and only one zero mode, $(\mathbf{3}, \bar{\mathbf{2}})_{Q12}$, from the chiral multiplet Σ_1 , which can be identified with the third-family quark doublet Q_3 .

B. $SU(6)$ model II and $SU(6)$ model III

In this subsection, we will construct $SU(6)$ models with gauge-top and gauge-bottom quark Yukawa coupling unification. We consider 6D $\mathcal{N} = (1, 1)$ supersymmetric $SU(6)$ compactified on the orbifold $M^4 \times T^2/Z_6$. To break the $SU(6)$ gauge symmetry, we choose the following 6×6 matrix representation for R_{Γ_T}

$$R_{\Gamma_T} = \text{diag}(+1, +1, +1, \omega^{n_1}, \omega^{n_1}, \omega^{n_2}), \quad (21)$$

where $n_1 \neq n_2 \neq 0$. Then, we obtain

$$SU(6)/R_{\Gamma_T} = SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_\alpha. \quad (22)$$

The Z_6 transformation properties for the decomposed components of V , Σ_1 , Σ_2 , and Σ_3 are given by the first 3×3 submatrices in Eqs. (A9)–(A12) in Appendix A. We choose

$$k = 1, \quad n_1 = 5, \quad n_2 = 2, \quad (23)$$

and consider the following two models:

- (A) $SU(6)$ model II (gauge-top quark Yukawa coupling unification)

With

$$a = \frac{1}{10}, \quad (24)$$

we have

$$k_Y = \frac{2}{3}, \quad (25)$$

The zero modes from the chiral multiplets Σ_1 , Σ_2 , and Σ_3 are presented in Table I. We can identify them as the third-family quark doublet, the right-handed top quark, and the MSSM Higgs doublets. From the trilinear term in the 6D bulk action, we obtain the top quark Yukawa term

$$\int d^6x \left[\int d^2\theta g_6 Q_3 t^c H_u + \text{H.c.} \right]. \quad (26)$$

Thus, at M_{GUT} , we have

$$g_1 = g_2 = g_3 = y_t = g_6/\sqrt{V}, \quad (27)$$

where y_t is the top quark Yukawa coupling, and V is the physical volume of extra dimensions.

- (B) $SU(6)$ model III (gauge-bottom quark Yukawa coupling unification)

For this case we set

$$a = -\frac{1}{15}, \quad (28)$$

in which case

$$k_Y = \frac{1}{3}, \quad (29)$$

The zero modes arise from the chiral multiplets Σ_1 , Σ_2 , and Σ_3 , and are presented in Table II. We can identify them as the third-family quark doublet, the right-handed bottom quark, and the MSSM Higgs

TABLE I. Zero modes from the chiral multiplets Σ_1 , Σ_2 , and Σ_3 in $SU(6)$ (Model II).

Chiral fields	Zero modes
Σ_1	$Q_3: (\mathbf{3}, \bar{\mathbf{2}})_{Q12}$
Σ_2	$t^c: (\bar{\mathbf{3}}, \mathbf{1})_{Q31}$
Σ_3	$H_u: (\mathbf{1}, \mathbf{2})_{Q23}; H_d: (\mathbf{1}, \bar{\mathbf{2}})_{Q32}$

TABLE II. Zero modes from the chiral multiplets Σ_1 , Σ_2 , and Σ_3 in $SU(6)$ (Model III).

Chiral fields	Zero modes
Σ_1	$Q_3: (\mathbf{3}, \bar{\mathbf{2}})_{Q12}$
Σ_2	$b^c: (\bar{\mathbf{3}}, \mathbf{1})_{Q31}$
Σ_3	$H_d: (\mathbf{1}, \mathbf{2})_{Q23}; H_u: (\mathbf{1}, \bar{\mathbf{2}})_{Q32}$

doublets.

From the trilinear term in the 6D bulk action, we obtain the bottom quark Yukawa term

$$\int d^6x \left[\int d^2\theta g_6 Q_3 b^c H_d + \text{H.c.} \right]. \quad (30)$$

Thus, at M_{GUT} , we have

$$g_1 = g_2 = g_3 = y_b = g_6/\sqrt{V}, \quad (31)$$

where y_b is the bottom quark Yukawa coupling.

IV. $SU(7)$ MODELS

As we discussed above, to achieve gauge-fermion-Higgs unification, the minimal gauge group is $SU(7)$, with $U(1)_Y$ normalization $k_Y = 23/21$ which is uniquely determined. This can be seen as follows. The $U(1)_Y$ generator in $SU(7)$ belongs to its Cartan subalgebra, and can be parametrized as

$$T_{U(1)_Y} \equiv \text{diag}(r_3, r_3, r_3, r_2, r_2, r_1, r_1'). \quad (32)$$

The traceless condition yields

$$3r_3 + 2r_2 + r_1 + r_1' = 0, \quad (33)$$

and gauge-fermion-Higgs unification requires that

$$r_3 - r_2 = \frac{1}{6}, \quad r_3 - r_1 = \frac{2}{3}, \quad r_3 - r_1' = -\frac{1}{3}. \quad (34)$$

Thus, we have the unique solution

$$r_3 = \frac{2}{21}, \quad r_2 = -\frac{1}{14}, \quad r_1 = -\frac{4}{7}, \quad r_1' = \frac{3}{7}, \quad (35)$$

for which $\text{tr}[T_{U(1)_Y}^2] = 23/42$. With a canonical normalization $\text{tr}[T_i^2] = 1/2$ of non-Abelian generators, we obtain $k_Y = 23/21$.

We consider a 6D $\mathcal{N} = (1, 1)$ supersymmetric $SU(7)$ gauge theory compactified on the orbifold $M^4 \times T^2/Z_6$ (for some details see Appendix A). To break $SU(7)$, we select the following 7×7 matrix representation for R_{Γ_T}

$$R_{\Gamma_T} = \text{diag}(+1, +1, +1, \omega^{n_1}, \omega^{n_1}, \omega^{n_2}, \omega^{n_3}), \quad (36)$$

where $n_1 \neq n_2 \neq n_3 \neq 0$. Thus,

$$SU(7)/R_{\Gamma_T} = SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_\beta \times U(1)_\gamma. \quad (37)$$

So, for the zero modes, the 6D $\mathcal{N} = (1, 1)$ supersymmetric $SU(7)$ gauge symmetry is broken down to 4D $\mathcal{N} = 1$ supersymmetric $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_\beta \times U(1)_\gamma$ gauge symmetry [2]. We assume that the two additional $U(1)$ symmetries can be spontaneously broken at M_{GUT} by the usual Higgs mechanism. It is conceivable that these two symmetries can play some useful role as flavor symmetries [8], but we will not pursue this any further here.

We define the generators for the $U(1)_Y \times U(1)_\beta \times U(1)_\gamma$ gauge symmetry as follows

$$\begin{aligned}
T_{U(1)_Y} &\equiv \frac{1}{42} \text{diag}(4, 4, 4, -3, -3, -24, 18), \\
T_{U(1)_\beta} &\equiv \text{diag}(1, 1, 1, 0, 0, -1, -2), \\
T_{U(1)_\gamma} &\equiv \text{diag}(3, 3, 3, -8, -8, 5, 2).
\end{aligned} \tag{38}$$

The $SU(7)$ adjoint representation **48** decomposes under the $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_\beta \times U(1)_\gamma$ gauge symmetry as

$$\begin{aligned}
\mathbf{48} = & \begin{pmatrix} (\mathbf{8}, \mathbf{1})_{Q00} & (\mathbf{3}, \bar{\mathbf{2}})_{Q12} & (\mathbf{3}, \mathbf{1})_{Q13} & (\mathbf{3}, \mathbf{1})_{Q14} \\ (\bar{\mathbf{3}}, \mathbf{2})_{Q21} & (\mathbf{1}, \mathbf{3})_{Q00} & (\mathbf{1}, \mathbf{2})_{Q23} & (\mathbf{1}, \mathbf{2})_{Q24} \\ (\bar{\mathbf{3}}, \mathbf{1})_{Q31} & (\mathbf{1}, \bar{\mathbf{2}})_{Q32} & (\mathbf{1}, \mathbf{1})_{Q00} & (\mathbf{1}, \mathbf{1})_{Q34} \\ (\bar{\mathbf{3}}, \mathbf{1})_{Q41} & (\mathbf{1}, \bar{\mathbf{2}})_{Q42} & (\mathbf{1}, \mathbf{1})_{Q43} & (\mathbf{1}, \mathbf{1})_{Q00} \end{pmatrix} \\
& + (\mathbf{1}, \mathbf{1})_{Q00},
\end{aligned} \tag{39}$$

where $(\mathbf{1}, \mathbf{1})_{Q00}$ in the third and fourth diagonal entries of the matrix and the last term $(\mathbf{1}, \mathbf{1})_{Q00}$ denote the gauge fields associated with $U(1)_Y \times U(1)_\beta \times U(1)_\gamma$. The subscripts Qij , which are antisymmetric ($Qij = -Qji$), are the charges under $U(1)_Y \times U(1)_\beta \times U(1)_\gamma$. The subscript $Q00 = (\mathbf{0}, \mathbf{0}, \mathbf{0})$, and the other subscripts Qij with $i \neq j$ are

$$\begin{aligned}
Q12 &= (\tfrac{1}{6}, \mathbf{1}, \mathbf{11}), & Q13 &= (\tfrac{2}{3}, \mathbf{2}, -2), \\
Q14 &= (-\tfrac{1}{3}, \mathbf{3}, \mathbf{1}), & Q23 &= (\tfrac{1}{2}, \mathbf{1}, -\mathbf{13}), \\
Q24 &= (-\tfrac{1}{2}, \mathbf{2}, -\mathbf{10}), & Q34 &= (-\mathbf{1}, \mathbf{1}, \mathbf{3}).
\end{aligned} \tag{40}$$

The Z_6 transformation properties for the decomposed components of V , Σ_1 , Σ_2 , and Σ_3 are given by Eqs. (A9)–(A12). We will consider two concrete models.

A. $SU(7)$ model I

We choose

$$k = 1, \quad n_1 = 4, \quad n_2 = 1, \quad n_3 = 2, \tag{41}$$

where k is given in Eqs. (A7) and (A8) in Appendix A. The zero modes from the chiral multiplets Σ_1 , Σ_2 , and Σ_3 are presented in Table III. We can identify them as the third-family SM fermions, and one pair of Higgs doublets. Interestingly, we do not have any exotic particle from the zero modes of the chiral multiplets.

From the trilinear term in the 6D bulk action, we obtain the top quark and tau lepton Yukawa terms

$$\int d^6x \left[\int d^2\theta g_7 (Q_3 t^c H_u + L_3 \tau^c H_d) + \text{H.c.} \right]. \tag{42}$$

TABLE III. Zero modes from the chiral multiplets Σ_1 , Σ_2 , and Σ_3 in $SU(7)$ (Model I).

Chiral fields	Zero modes
Σ_1	$t^c: (\bar{\mathbf{3}}, \mathbf{1})_{Q31}; \tau^c: (\mathbf{1}, \mathbf{1})_{Q43}$
Σ_2	$Q_3: (\mathbf{3}, \bar{\mathbf{2}})_{Q12}; H_d: (\mathbf{1}, \mathbf{2})_{Q24}; b^c: (\bar{\mathbf{3}}, \mathbf{1})_{Q41}$
Σ_3	$H_u: (\mathbf{1}, \mathbf{2})_{Q23}; L_3: (\mathbf{1}, \bar{\mathbf{2}})_{Q32}$

Thus, at M_{GUT} , we have

$$g_1 = g_2 = g_3 = y_t = y_\tau = g_7/\sqrt{V}, \tag{43}$$

where y_τ is the tau lepton Yukawa coupling. However, we do not have the bottom quark Yukawa term from 6D bulk action.

B. $SU(7)$ model II

We choose

$$k = 1, \quad n_1 = 4, \quad n_2 = 1, \quad n_3 = 3. \tag{44}$$

The zero modes from the chiral multiplets Σ_1 , Σ_2 , and Σ_3 are given in Table IV. We can identify them as the third-family SM fermions, the MSSM Higgs doublets, and an exotic (left-handed singlet) quark b_X .

From the trilinear term in the 6D bulk action, we obtain the top quark, bottom quark, and tau lepton Yukawa terms

$$\int d^6x \left[\int d^2\theta g_7 (Q_3 t^c H_u + Q_3 b^c H_d + L_3 \tau^c H_d) + \text{H.c.} \right]. \tag{45}$$

Thus, at M_{GUT} , we have

$$g_1 = g_2 = g_3 = y_t = y_b = y_\tau = g_7/\sqrt{V}. \tag{46}$$

Thus, we have unification of the SM gauge couplings and the third-family SM fermion Yukawa couplings.

We can give GUT-scale mass to the exotic quark b_X by introducing an additional exotic quark \bar{b}_X with quantum number $(\bar{\mathbf{3}}, \mathbf{1})_{QX}$ on the observable 3-brane at $z = 0$, where $QX = (\frac{1}{3}, -\mathbf{3}, \mathbf{0})$. Suppose we introduce one pair of SM singlets S' and \bar{S}' with charges $\mathbf{1}$ and $-\mathbf{1}$, respectively, whose VEVs break $U(1)_\gamma$ at M_{GUT} . The exotic quarks b_X and \bar{b}_X can pair up and acquire M_{GUT} mass via the brane-localized superpotential term $\bar{S}' b_X \bar{b}_X$.

V. $SU(8)$ MODELS

We are unable to construct orbifold models of gauge-fermion-Higgs unification with $k_Y < 23/21$. To construct models with $k_Y \geq 23/21$, we consider a 7D $\mathcal{N} = 1$ supersymmetric $SU(8)$ gauge theory compactified on the orbifold $M^4 \times T^2/Z_6 \times S^1/Z_2$ (for some details see Appendix B). To break the $SU(8)$ gauge symmetry, we choose the following 8×8 matrix representations for R_{Γ_7} and R_{Γ_5}

TABLE IV. Zero modes from the chiral multiplets Σ_1 , Σ_2 , and Σ_3 in $SU(7)$ (Model II).

Chiral fields	Zero modes
Σ_1	$H_d: (\mathbf{1}, \mathbf{2})_{Q24}; t^c: (\bar{\mathbf{3}}, \mathbf{1})_{Q31}$
Σ_2	$Q_3: (\mathbf{3}, \bar{\mathbf{2}})_{Q12}; \tau^c: (\mathbf{1}, \mathbf{1})_{Q43}$
Σ_3	$H_u: (\mathbf{1}, \mathbf{2})_{Q23}; L_3: (\mathbf{1}, \bar{\mathbf{2}})_{Q32}; b^c: (\bar{\mathbf{3}}, \mathbf{1})_{Q41}; b_X: (\mathbf{3}, \mathbf{1})_{Q14}$

$$R_{\Gamma_T} = \text{diag}(+1, +1, +1, \omega^{n_1}, \omega^{n_1}, \omega^{n_1}, +1, \omega^{n_2}), \quad (47)$$

$$R_{\Gamma_S} = \text{diag}(+1, +1, +1, +1, +1, -1, -1, +1), \quad (48)$$

where $n_1 \neq n_2 \neq 0$. We obtain

$$SU(8)/R_{\Gamma_T} = SU(4) \times SU(3) \times U(1)^2, \quad (49)$$

$$SU(8)/R_{\Gamma_S} = SU(6) \times SU(2) \times U(1), \quad (50)$$

$$SU(8)/\{R_{\Gamma_T} \cup R_{\Gamma_S}\} = SU(3)_C \times SU(2)_L \times U(1)_Y \\ \times U(1)_\alpha \times U(1)_\beta \times U(1)_\gamma. \quad (51)$$

Thus, for the zero modes, the 7D $\mathcal{N} = 1$ supersymmetric $SU(8)$ gauge symmetry is broken down to a 4D $\mathcal{N} = 1$ supersymmetric $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_\alpha \times U(1)_\beta \times U(1)_\gamma$ gauge symmetry [2].

We define the generators for the $U(1)_Y \times U(1)_\alpha \times U(1)_\beta \times U(1)_\gamma$ gauge symmetry as follows:

$$T_{U(1)_Y} \equiv \frac{1}{42} \text{diag}(4, 4, 4, -3, -3, -24, 18, 0) \\ + a \text{diag}(1, 1, 1, 1, 1, 1, -7),$$

$$T_{U(1)_\alpha} \equiv \text{diag}(4, 4, 4, -3, -3, -24, 18, 0) \\ - \frac{23}{56a} \text{diag}(1, 1, 1, 1, 1, 1, -7), \quad (52)$$

$$T_{U(1)_\beta} \equiv \text{diag}(1, 1, 1, 0, 0, -1, -2, 0),$$

$$T_{U(1)_\gamma} \equiv \text{diag}(3, 3, 3, -8, -8, 5, 2, 0),$$

where a is a real number. Because $\text{tr}[T_{U(1)_Y}^2] = 23/42 +$

$$Q_{12} = \left(\frac{1}{6}, 7, 1, 11\right), \quad Q_{13} = \left(\frac{2}{3}, 28, 2, -2\right), \quad Q_{14} = \left(-\frac{1}{3}, -14, 3, 1\right), \quad Q_{23} = \left(\frac{1}{2}, 21, 1, -13\right), \\ Q_{24} = \left(-\frac{1}{2}, -21, 2, -10\right), \quad Q_{34} = (-1, -42, 1, 3), \quad Q_{15} = \left(\frac{2}{21} + 8a, 4 - \frac{23}{7a}, 1, 3\right), \quad (55)$$

$$Q_{25} = \left(-\frac{1}{14} + 8a, -3 - \frac{23}{7a}, 0, -8\right), \quad Q_{35} = \left(-\frac{4}{7} + 8a, -24 - \frac{23}{7a}, -1, 5\right), \quad Q_{45} = \left(\frac{3}{7} + 8a, 18 - \frac{23}{7a}, -2, 2\right).$$

The $Z_6 \times Z_2$ transformation properties for the decomposed components of V , Σ_1 , Σ_2 , and Σ_3 are given by Eqs. (B12)–(B15) in Appendix B. And we choose

$$n_1 = 5, \quad n_2 = 2, \quad \text{or } 3. \quad (56)$$

The zero modes from the chiral multiplets Σ_1 , Σ_2 , and Σ_3 are presented in Table V. We can identify them as the third-family SM fermions, the MSSM Higgs doublets, and the exotic quark b_X .

From the trilinear term in the 7D bulk action, we obtain the top quark, bottom quark, and tau lepton Yukawa terms

$$\int d^7x \left[\int d^2\theta g_8 (Q_3 t^c H_u + Q_3 b^c H_d + L_3 \tau^c H_d) + \text{H.c.} \right]. \quad (57)$$

$56a^2$, we obtain

$$k_Y = \frac{23}{21} + 112a^2 \geq \frac{23}{21}. \quad (53)$$

Incidentally, for the canonical $U(1)_Y$ normalization ($k_Y = 5/3$), we have $a = 1/14$, and $U(1)_Y$ coincides with $U(1)_Y$ in the Pati-Salam or Pati-Salam-like models when we break $SU(8)$ down to $SU(4)_C \times SU(2)_L \times SU(2)_R \times U(1)^2$ or $SU(3)_C \times SU(2)_L \times U(1)_{B-L} \times U(1)_{I_{3R}} \times U(1)^2$ by orbifold projections.

The $SU(8)$ adjoint representation **63** decomposes under $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_\alpha \times U(1)_\beta \times U(1)_\gamma$ gauge symmetry as:

$$\mathbf{63} = \begin{pmatrix} (\mathbf{8}, \mathbf{1})_{Q00} & (\mathbf{3}, \bar{\mathbf{2}})_{Q12} & (\mathbf{3}, \mathbf{1})_{Q13} & (\mathbf{3}, \mathbf{1})_{Q14} & (\mathbf{3}, \mathbf{1})_{Q15} \\ (\bar{\mathbf{3}}, \mathbf{2})_{Q21} & (\mathbf{1}, \mathbf{3})_{Q00} & (\mathbf{1}, \mathbf{2})_{Q23} & (\mathbf{1}, \mathbf{2})_{Q24} & (\mathbf{1}, \mathbf{2})_{Q25} \\ (\bar{\mathbf{3}}, \mathbf{1})_{Q31} & (\mathbf{1}, \bar{\mathbf{2}})_{Q32} & (\mathbf{1}, \mathbf{1})_{Q00} & (\mathbf{1}, \mathbf{1})_{Q34} & (\mathbf{1}, \mathbf{1})_{Q35} \\ (\bar{\mathbf{3}}, \mathbf{1})_{Q41} & (\mathbf{1}, \bar{\mathbf{2}})_{Q42} & (\mathbf{1}, \mathbf{1})_{Q43} & (\mathbf{1}, \mathbf{1})_{Q00} & (\mathbf{1}, \mathbf{1})_{Q45} \\ (\bar{\mathbf{3}}, \mathbf{1})_{Q51} & (\mathbf{1}, \bar{\mathbf{2}})_{Q52} & (\mathbf{1}, \mathbf{1})_{Q53} & (\mathbf{1}, \mathbf{1})_{Q54} & (\mathbf{1}, \mathbf{1})_{Q00} \end{pmatrix} \\ + (\mathbf{1}, \mathbf{1})_{Q00}, \quad (54)$$

where $(\mathbf{1}, \mathbf{1})_{Q00}$ in the third, fourth, and fifth diagonal entries of the matrix, and the last term $(\mathbf{1}, \mathbf{1})_{Q00}$ denote the gauge fields for $U(1)_Y \times U(1)_\alpha \times U(1)_\beta \times U(1)_\gamma$. Moreover, the subscripts Qij , with $Qij = -Qji$, are the charges under $U(1)_Y \times U(1)_\alpha \times U(1)_\beta \times U(1)_\gamma$. The subscript $Q00 = (\mathbf{0}, \mathbf{0}, \mathbf{0}, \mathbf{0})$, and the other subscripts Qij with $i \neq j$ are

Thus, at M_{GUT} , we have

$$g_1 = g_2 = g_3 = y_t = y_b = y_\tau = g_8/\sqrt{V}. \quad (58)$$

TABLE V. Zero modes from the chiral multiplets Σ_1 , Σ_2 , and Σ_3 in the $SU(8)$ model.

Chiral fields	Zero modes
Σ_1	$Q_3: (\mathbf{3}, \bar{\mathbf{2}})_{Q12}; \tau^c: (\mathbf{1}, \mathbf{1})_{Q43};$
Σ_2	$H_u: (\mathbf{1}, \mathbf{2})_{Q23}; L_3: (\mathbf{1}, \bar{\mathbf{2}})_{Q32}; b^c: (\bar{\mathbf{3}}, \mathbf{1})_{Q41}; b_X: (\mathbf{3}, \mathbf{1})_{Q14}$
Σ_3	$H_d: (\mathbf{1}, \mathbf{2})_{Q24}; t^c: (\bar{\mathbf{3}}, \mathbf{1})_{Q31}$

VI. NEW PARTICLES AND GAUGE COUPLING UNIFICATION

For noncanonical $U(1)_Y$ normalization, it is necessary to introduce new particles to achieve unification. Here, as an example, we consider restoring gauge coupling unification by adding a minimal set of vectorlike particles with SM quantum numbers. These particles can be put on the 3-brane at $z = 0$, and their masses can be the order of the weak scale due to the Giudice-Masiero mechanism [9].

We denote these particles as u_x and so on, where u_x stands for the vectorlike pair with the same quantum numbers as these for $u + u^c$. Although we employ two-loop renormalization group equations (RGEs) for the gauge couplings in the numerical calculations, for the discussions below we will consider one-loop β -coefficients which, for the MSSM and vectorlike particles, are as follows:

$$\begin{aligned} b_{\text{MSSM}} &= \left(\frac{11}{k_Y}, 1, -3 \right), & b_{Q_x} &= \left(\frac{1}{3k_Y}, 3, 2 \right), \\ b_{u_x} &= \left(\frac{8}{3k_Y}, 0, 1 \right), & b_{d_x} &= \left(\frac{2}{3k_Y}, 0, 1 \right), \\ b_{L_x} &= \left(\frac{1}{k_Y}, 1, 0 \right), & b_{e_x} &= \left(\frac{2}{k_Y}, 0, 0 \right). \end{aligned} \quad (59)$$

From the one-loop RGEs, it is straightforward to obtain the following relations:

$$\log \frac{M_{\text{GUT}}}{m_Z} = \frac{2\pi(\alpha_s^{-1}\alpha - s_W^2)}{\alpha(b_3 - b_2)}, \quad (60)$$

$$\alpha_s^{-1} = \left[s_W^2 + \frac{1 - (1 + k_Y)s_W^2}{k_Y} \left(\frac{b_3 - b_2}{b_1 - b_2} \right) \right] \alpha^{-1}, \quad (61)$$

$$\alpha_{\text{GUT}} = \frac{k_Y \alpha (b_1 - b_2)}{k_Y s_W^2 b_1 - (1 - s_W^2) b_2}, \quad (62)$$

where s_W stands for $\sin\theta_W$, and α and α_s are the electromagnetic and strong couplings at m_Z . From Eq. (59), we see that $b_3 - b_2$ is an integer. For the GUT scale to be smaller than the Planck scale and large enough to avoid the bounds on proton decay, Eq. (60) requires the contribution $(b_3 - b_2)_x$ from vectorlike particles to vanish, assuming the latter have masses close to the weak scale. From Eq. (61), the range of $(b_1 - b_2)_x$ allowing gauge coupling unification can be obtained depending on the value of k_Y . Also, $\alpha_{\text{GUT}} \ll 1$ is required for perturbative unification.

Simple examples that satisfy the above conditions are as follows. For $k_Y = 1/15$ as in the $SU(5)$ model, gauge coupling unification can be restored by adding two sets of $L_x + u_x$. Unification can also be restored by adding $L_x + u_x + 2e_x$ or by adding $4e_x$. For $k_Y = 1/3$ as in the $SU(6)$ model with gauge-bottom quark Yukawa coupling unification, one can again add two sets of $L_x + u_x$, or $3e_x$. And for $k_Y = 2/3$ as in the $SU(6)$ model with gauge-top

quark Yukawa coupling unification, one can add $L_x + u_x + e_x$ or $3(L_x + d_x) + e_x$. Finally, for $k_Y = 23/21$ as in the $SU(7)$ model with the unification of the gauge couplings and third-family Yukawa couplings, one can add $L_x + u_x$. Because such additional vectorlike particles can be observed at the LHC and ILC, we can distinguish these models with these future experiments.

VII. NEW PARTICLES AND GAUGE-YUKAWA UNIFICATION

In this section we probe gauge-Yukawa unification following the analysis in Ref. [10] (see also Ref. [11] for details and references). In our analysis, we use a dimensional reduction ($\overline{\text{DR}}$) renormalization scheme, which is known to be consistent with SUSY $\overline{\text{DR}}$ Yukawa couplings ($y_{l,b,\tau}$) and gauge couplings (g_i) in the MSSM at Z-boson mass scale are written as follows:

$$y_l(m_Z) = \frac{\sqrt{2}\bar{m}_l^{\text{MSSM}}(m_Z)}{\bar{v}(m_Z)\sin\beta} = \frac{\sqrt{2}\bar{m}_l^{\text{SM}}(m_Z)}{\bar{v}(m_Z)\sin\beta}(1 + \delta_l), \quad (63)$$

$$y_{b,\tau}(m_Z) = \frac{\sqrt{2}\bar{m}_{b,\tau}^{\text{MSSM}}(m_Z)}{\bar{v}(m_Z)\cos\beta} = \frac{\sqrt{2}\bar{m}_{b,\tau}^{\text{SM}}(m_Z)}{\bar{v}(m_Z)\cos\beta}(1 + \delta_{b,\tau}), \quad (64)$$

$$g_i(m_Z) = \bar{g}_i^{\text{SM}}(m_Z)(1 + \delta_{g_i}), \quad (i = 1-3), \quad (65)$$

where \bar{m}_i^{SM} and \bar{g}_i^{SM} are $\overline{\text{DR}}$ quantities defined in the SM, and \bar{v} and $\tan\beta$ are $\overline{\text{DR}}$ values in the MSSM. They are determined following the analysis in Ref. [11]. We adopt top pole mass ($m_t = 172.5$ GeV) [12], tau pole mass ($m_\tau = 1777$ MeV), $\overline{\text{MS}}$ bottom mass ($\bar{m}_b^{\text{MS}}(\bar{m}_b^{\text{MS}}) = 4.26$ GeV), and $\alpha_s^{\text{MS}}(m_Z) = 0.119$ [13]. The quantities δ_{l,b,τ,g_i} represent SUSY threshold corrections. In our analysis, we treat them as free parameters without specifying any particular SUSY breaking scenario. When all parameters δ_{l,b,τ,g_i} are specified, all $\overline{\text{DR}}$ couplings in the MSSM are determined at m_Z . Then we use the two-loop RGEs for the MSSM gauge couplings and the one-loop RGEs for the Yukawa couplings in order to study the unification of couplings at the GUT scale.

In order to study the gauge-Yukawa unification, we look for a region where top, bottom, and tau Yukawa couplings are unified ($y_t = y_b = y_\tau \equiv y_G$) at the GUT scale. We define the GUT scale (M_G) as a scale where $g_1(M_G) = g_2(M_G) \equiv g_G$. In our analysis, we allow the possibility that the strong gauge coupling is not exactly unified, i.e., $g_3(M_G)^2/g_G^2 = 1 + \epsilon_3$ where ϵ_3 can be a few %. This mismatch ϵ_3 from exact unification can be due to the GUT-scale threshold corrections to the unified gauge coupling.

First, we review gauge-Yukawa unification for the canonical case $k_Y = 5/3$. In Fig. 1, contours of δ_b (dotted lines in Fig. 1(a)), $\tan\beta$ (dashed lines in Fig. 1(b)), and ϵ_3 (dotted lines in Fig. 1(b)) are shown as a function of δ_l and

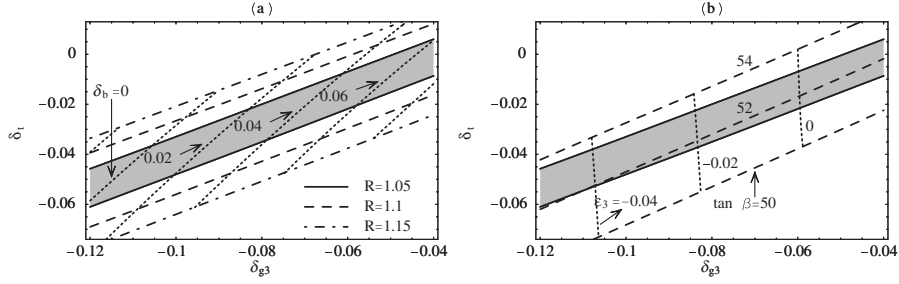


FIG. 1. Parameter space satisfying the gauge-Yukawa unification. Contours of δ_b (dotted lines in Fig. 1(a)), $\tan\beta$ (dashed lines in Fig. 1(b)), and ϵ_3 (dotted lines in Fig. 1(b)) are shown as a function of δ_t and δ_{g_3} , required for Yukawa unification ($y_t = y_b = y_\tau$). After finding the region for the Yukawa unification, contours of a parameter R (defined in text) are plotted in Fig. 1(a). The shaded regions represent a region where the gauge-Yukawa unification is achieved within 5% level ($R \leq 1.05$). Here we have fixed $\delta_\tau = 0.02$, $\delta_{g_1} = -0.006$, and $\delta_{g_2} = -0.02$.

δ_{g_3} , which are required for the Yukawa unification at the GUT scale. In order to fix $\delta_{g_{1,2}}$, we assume that all SUSY mass parameters which contribute to $\delta_{g_{1,2}}$ are equal to 500 GeV ($\delta_{g_1} = -0.006$ and $\delta_{g_2} = -0.02$). As shown in Fig. 1, $\tan\beta$ should be about 52, and the value of δ_b should be a few %, which is much smaller than 1 naively expected in large $\tan\beta$ case. Small values of δ_b significantly constrains the superpartner spectrum, as discussed in Refs. [11,14]. On the other hand, δ_t is in the expected range (see Ref. [15]).

After requiring Yukawa unification, we calculate a parameter R defined as follows:

$$R \equiv \frac{\max(y_G, g_G)}{\min(y_G, g_G)}. \quad (66)$$

In the shaded regions of Fig. 1, gauge-Yukawa unification is realized within 5% level ($R \leq 1.05$), while allowing ϵ_3 to be a few %.

Next, we take $k_Y = 23/21$ as predicted by the $SU(7)$ model, and give examples as to how gauge-Yukawa unification might be realized. Gauge coupling unification can be restored by adding vectorlike particles with SM quantum numbers, as in Sec. VI. A simple example for $k_Y = 23/21$ is adding one set of $L_x + u_x$. However, as shown in Fig. 2, Yukawa unification then requires δ_t shifted up 0.06 compared to Fig. 1, which is not compatible with the SUSY threshold corrections in most of the parameter space.

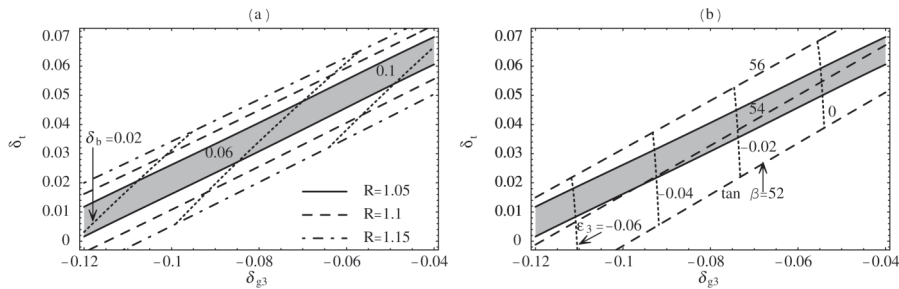


FIG. 2. Same as Fig. 1, but for $k_Y = 23/21$ with one set of $L_x + u_x$ added at $M = 300$ GeV.

Note that δ_t can be modified if mixing in the top quark sector is allowed. We then have the Yukawa and mass terms

$$y_t Q' H_u u'^c + y' Q' H_u u'_x{}^c + M u'_x u'_x{}^c, \quad (67)$$

where the primes denote weak eigenstates. Diagonalizing the mass matrix, we obtain

$$\frac{y_t}{y_{t0}} = \left(\frac{2}{1 + \xi^2 + x^2 - \sqrt{(1 + \xi^2 + x^2)^2 - 4x^2}} \right)^{1/2}. \quad (68)$$

Here the notation is as follows: y_{t0} is the value without mixing, $x \equiv M/m_t$, and $\xi \equiv y'/y_t$. Experimentally, $M \leq 200$ GeV is excluded [16]. As an example we take $M = 300$ GeV. Precision electroweak data (more precisely the bounds on the oblique parameter T) then requires the extended Cabibbo-Kobayashi-Maskawa (CKM) parameter $V_{xb} \leq 0.4$ [17]. This constraint corresponds to $\xi \leq 0.5$ and a downward shift in δ_t of ≤ 0.06 .

A similar example is adding one set of $L_x + d_x + e_x$. Gauge-Yukawa unification is then obtained essentially with the same parameters as above, since the β -coefficients are identical at one loop. δ_t in this case can be modified even assuming no mixing, due to the new Yukawa couplings $y_1 L_x H_d e_x^c + y_2 L_x^c H_u e_x$. Shifting δ_t down appreciably requires no or a weak y_1 coupling and a strong y_2 coupling, and a numerical example is provided in Fig. 3.

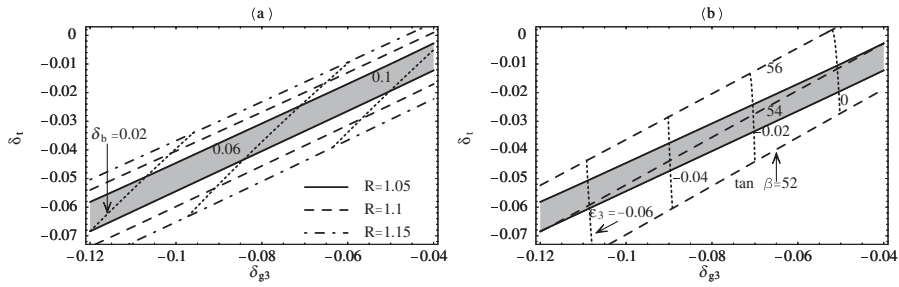


FIG. 3. Same as Fig. 1, but for $k_Y = 23/21$ with one set of $L_x + d_x + e_x$ added at $M = 300$ GeV. The Yukawa coupling y_1 is assumed negligible, while y_2 is taken to be 0.7 at M , corresponding to ≈ 1.5 at the GUT scale.

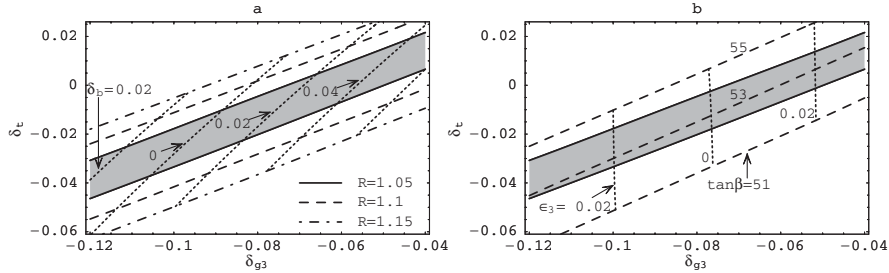


FIG. 4. Same as Fig. 1, but for $k_Y = 23/21$ with vectorlike charged singlets (one pair with $Q = \pm 1$ and one pair with $Q = \pm 2/3$) added at m_Z .

Another way to restore gauge coupling unification while preserving Yukawa unification is to add vectorlike charged singlets and allow fractional charges. As an example, we again take $k_Y = 23/21$, and add two pairs of charged singlets with mass m_Z and charges ± 1 and $\pm 2/3$. As shown in Fig. 4, gauge-Yukawa unification is then achieved similar to the canonical case.

In Fig. 5, we show the charge of a vectorlike charged singlet pair with mass m_Z allowing unification, for k_Y in the range $1/15$ to $5/3$. (Adding one pair with charges $\pm Q$ is equivalent at one-loop to adding multiple pairs with charges $\pm Q_i$ if $Q^2 = \sum_i Q_i^2$.) Here we choose δ_{t,b,τ,g_i} such that $Q = 0$ for $k_Y = 5/3$ and $\alpha_s^{MS}(m_Z) = 0.119$.

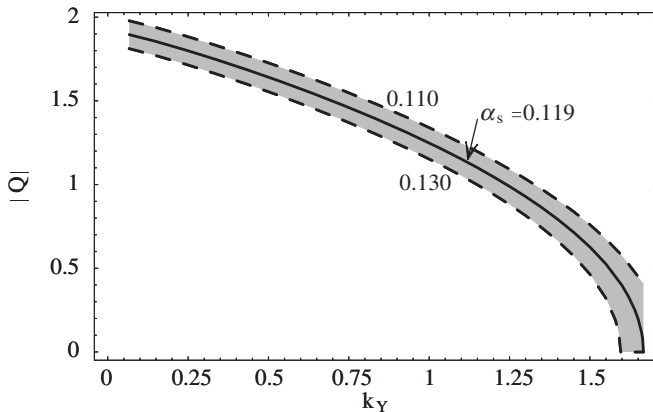


FIG. 5. $|Q|$ of the vectorlike charged singlet with mass m_Z allowing unification for k_Y in the range $1/15$ to $5/3$.

The ± 0.01 uncertainty we display for $\alpha_s^{MS}(m_Z)$ represents both SUSY and GUT threshold corrections.

For fractionally charged singlets, there is a constraint on particle per nucleon of about 10^{-22} [18]. This requires the particle mass M to be $\geq 10^4 T_r$, where T_r is the reheating temperature [19].² Since T_r can be as low as a few MeV, this in principle allows fractionally charged singlets as light as allowed by accelerator searches. The mass limit from accelerators is around m_Z (for a review see Ref. [20]).

VIII. HIGGS MASS

We end the paper with some remarks on the Higgs mass, where by the Higgs mass we refer to the mass of the light CP -even scalar. Assuming that $m_Z \ll m_{SUSY}$, where m_{SUSY} is the characteristic supersymmetry particle mass scale, the theory below m_{SUSY} is the SM with threshold effects at m_{SUSY} . The SM Higgs quartic coupling at m_{SUSY} is given by

$$\lambda = \frac{1}{4}(g_Y^2 + g_2^2)\cos^2 2\beta = \frac{1}{4}\left(\frac{g_2^2}{\cos^2 \theta_W}\right)\cos^2 2\beta, \quad (69)$$

where $\tan \beta$ is the ratio of the two supersymmetric Higgs

²Since the fractionally charged particle is not neutralized it may have difficulty getting past the heliopause if is not accelerated by astrophysical processes. This may reduce the abundance on earth a few orders of magnitude, but since the abundance is very sensitive to M/T_r , the conclusion does not change much, and conservatively we can say $M \geq 10^3 T_r$.

vacuum expectation values, and θ_W is the Weinberg angle. Since $\cos^2\theta_W = k_Y/(1+k_Y)$ at M_{GUT} , θ_W at m_{SUSY} depends on k_Y . The Higgs mass therefore also depends on the value of k_Y , but for SUSY broken at the TeV scale the effect is numerically insignificant, of order a few hundred MeV. The Higgs mass predictions are therefore practically the same as in canonical MSSM [21] and SUSY $SO(10)$ for the case with third-family Yukawa unification [14,22]. The Higgs mass upper bound for $m_t = 172.5$ GeV and $m_{\text{SUSY}} = 1$ TeV is ≈ 130 GeV [21].

IX. CONCLUSION

We have considered a class of orbifold GUTs based on 6D $\mathcal{N} = (1, 1)$ and 7D $\mathcal{N} = 1$ supersymmetric $SU(N)$ gauge theories, where the 4D gauge group is $SU(3)_C \times SU(2)_L \times U(1)_Y$ below the compactification scale. For the $SU(5)$ model the only zero mode that can be introduced in the bulk is a quark doublet, while the $SU(6)$ model allows gauge-Higgs unification. Finally, we can have gauge-Yukawa unification for the third family in $SU(7)$ or higher rank groups. Depending on the model, the $U(1)_Y$ normalization factor k_Y is either uniquely determined to have a noncanonical value or lies in a continuous interval. Gauge coupling unification and gauge-Yukawa unification can be obtained for noncanonical k_Y values by adding particles to the MSSM spectrum. As examples, we introduce a minimal set of vectorlike multiplets with SM quantum numbers or fractionally charged color singlets, assuming masses in the TeV range. The existence of such particles will be tested by the upcoming LHC.

ACKNOWLEDGMENTS

This work is supported in part by DOE Grant No. DE-FG02-84ER40163 (I.G.), No. DE-FG02-96ER40959 (T.L.), No. DE-FG02-91ER40626 (Q.S. and V.N.S.), and by the University of Delaware (V.N.S.).

APPENDIX A: SIX-DIMENSIONAL ORBIFOLD MODELS

We consider 6D space-time which can be factorized into a product of 4D Minkowski space-time M^4 and the torus T^2 which is homeomorphic to $S^1 \times S^1$. The 6D coordinates are x^μ , ($\mu = 0, 1, 2, 3$), x^5 , and x^6 . The radii for the circles along the x^5 and x^6 directions are R_1 and R_2 , respectively. We define the complex coordinate

$$z \equiv \frac{1}{2}(x^5 + ix^6), \quad (\text{A1})$$

in which case the torus T^2 can be defined as C^1 modulo the equivalence classes:

$$z \sim z + \pi R_1, \quad z \sim z + \pi R_2 e^{i\theta}. \quad (\text{A2})$$

To define the orbifold T^2/Z_6 , we require that $R_1 = R_2 \equiv R$ and $\theta = \pi/3$. Then T^2/Z_6 orbifold is obtained from T^2 as:

$$\Gamma_T: z \sim \omega z, \quad (\text{A3})$$

where $\omega = e^{i\pi/3}$. There is one Z_6 fixed point: $z = 0$, two Z_3 fixed points: $z = \pi R e^{i\pi/6}/\sqrt{3}$ and $z = 2\pi R e^{i\pi/6}/\sqrt{3}$, and three Z_2 fixed points: $z = \sqrt{3}\pi R e^{i\pi/6}/2$, $z = \pi R/2$, and $z = \pi R e^{i\pi/3}/2$.

The $\mathcal{N} = (1, 1)$ supersymmetry in 6D has 16 supercharges and corresponds to $\mathcal{N} = 4$ supersymmetry in 4D, so that only the gauge multiplet can be introduced in the bulk. This multiplet can be decomposed under 4D $\mathcal{N} = 1$ supersymmetry into a vector multiplet V and three chiral multiplets Σ_1 , Σ_2 , and Σ_3 in the adjoint representation, where the fifth and sixth components of the gauge field, A_5 and A_6 are contained in the lowest component of Σ_1 . The SM fermions can be on the 3-branes at the Z_6 fixed points. Here, we follow the conventions in Ref. [23].

For the bulk gauge group G , we write down the bulk action in the Wess-Zumino gauge and 4D $\mathcal{N} = 1$ supersymmetry language [24],

$$\begin{aligned} \mathcal{S} = \int d^6x & \left\{ \text{Tr} \left[\int d^2\theta \left(\frac{1}{4\kappa g^2} \mathcal{W}^\alpha \mathcal{W}_\alpha \right. \right. \right. \\ & + \frac{1}{\kappa g^2} \left(\Sigma_3 \partial \Sigma_2 - \frac{1}{\sqrt{2}} \Sigma_1 [\Sigma_2, \Sigma_3] \right) \left. \left. \right) \right] + \text{H.c.} \left. \right\} \\ & + \int d^4\theta \frac{1}{\kappa g^2} \text{Tr} [(\sqrt{2} \partial_z^\dagger + \Sigma_1^\dagger) e^{-V} (-\sqrt{2} \partial_z + \Sigma_1) e^V \\ & + \partial_z^\dagger e^{-V} \partial_z e^V] + \int d^4\theta \frac{1}{\kappa g^2} \text{Tr} [\Sigma_2^\dagger e^{-V} \Sigma_2 e^V \\ & + \Sigma_3^\dagger e^{-V} \Sigma_3 e^V] \left. \right\}, \quad (\text{A4}) \end{aligned}$$

where κ is the normalization of the group generator, and \mathcal{W}_α denotes the gauge field strength. From the above action, we obtain the transformations of the vector multiplet

$$V(x^\mu, \omega z, \omega^{-1} \bar{z}) = R_{\Gamma_T} V(x^\mu, z, \bar{z}) R_{\Gamma_T}^{-1}, \quad (\text{A5})$$

$$\Sigma_1(x^\mu, \omega z, \omega^{-1} \bar{z}) = \omega^{-1} R_{\Gamma_T} \Sigma_1(x^\mu, z, \bar{z}) R_{\Gamma_T}^{-1}, \quad (\text{A6})$$

$$\Sigma_2(x^\mu, \omega z, \omega^{-1} \bar{z}) = \omega^{-1-k} R_{\Gamma_T} \Sigma_2(x^\mu, z, \bar{z}) R_{\Gamma_T}^{-1}, \quad (\text{A7})$$

$$\Sigma_3(x^\mu, \omega z, \omega^{-1} \bar{z}) = \omega^{2+k} R_{\Gamma_T} \Sigma_3(x^\mu, z, \bar{z}) R_{\Gamma_T}^{-1}, \quad (\text{A8})$$

where R_{Γ_T} is nontrivial to break the bulk gauge group G . To preserve 4D $\mathcal{N} = 1$ supersymmetry, we obtain $k = 0$ or $k = 1$ [23].

The Z_6 transformation properties for the decomposed components of V , Σ_1 , Σ_2 , and Σ_3 in the $SU(7)$ models are

$$V: \begin{pmatrix} 1 & \omega^{-n_1} & \omega^{-n_2} & \omega^{-n_3} \\ \omega^{n_1} & 1 & \omega^{n_1-n_2} & \omega^{n_1-n_3} \\ \omega^{n_2} & \omega^{n_2-n_1} & 1 & \omega^{n_2-n_3} \\ \omega^{n_3} & \omega^{n_3-n_1} & \omega^{n_3-n_2} & 1 \end{pmatrix} + 1, \quad (\text{A9})$$

$$\Sigma_1: \begin{pmatrix} \omega^{-1} & \omega^{-n_1-1} & \omega^{-n_2-1} & \omega^{-n_3-1} \\ \omega^{n_1-1} & \omega^{-1} & \omega^{n_1-n_2-1} & \omega^{n_1-n_3-1} \\ \omega^{n_2-1} & \omega^{n_2-n_1-1} & \omega^{-1} & \omega^{n_2-n_3-1} \\ \omega^{n_3-1} & \omega^{n_3-n_1-1} & \omega^{n_3-n_2-1} & \omega^{-1} \end{pmatrix} + \omega^{-1}, \quad (\text{A10})$$

$$\Sigma_2: \begin{pmatrix} \omega^{-1-k} & \omega^{-n_1-1-k} & \omega^{-n_2-1-k} & \omega^{-n_3-1-k} \\ \omega^{n_1-1-k} & \omega^{-1-k} & \omega^{n_1-n_2-1-k} & \omega^{n_1-n_3-1-k} \\ \omega^{n_2-1-k} & \omega^{n_2-n_1-1-k} & \omega^{-1-k} & \omega^{n_2-n_3-1-k} \\ \omega^{n_3-1-k} & \omega^{n_3-n_1-1-k} & \omega^{n_3-n_2-1-k} & \omega^{-1-k} \end{pmatrix} + \omega^{-1-k}, \quad (\text{A11})$$

$$\Sigma_3: \begin{pmatrix} \omega^{2+k} & \omega^{-n_1+2+k} & \omega^{-n_2+2+k} & \omega^{-n_3+2+k} \\ \omega^{n_1+2+k} & \omega^{2+k} & \omega^{n_1-n_2+2+k} & \omega^{n_1-n_3+2+k} \\ \omega^{n_2+2+k} & \omega^{n_2-n_1+2+k} & \omega^{2+k} & \omega^{n_2-n_3+2+k} \\ \omega^{n_3+2+k} & \omega^{n_3-n_1+2+k} & \omega^{n_3-n_2+2+k} & \omega^{2+k} \end{pmatrix} + \omega^{2+k}, \quad (\text{A12})$$

where the zero modes transform as 1. Note that $n_1 \neq n_2 \neq n_3 \neq 0$ and from Eqs. (A9)–(A12), we obtain that for the zero modes, the 6D $\mathcal{N} = (1, 1)$ supersymmetric $SU(7)$ gauge symmetry is broken down to 4D $\mathcal{N} = 1$ supersymmetric $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_\beta \times U(1)_\gamma$ gauge symmetry.

APPENDIX B: SEVEN-DIMENSIONAL ORBIFOLD MODELS

We consider a 7D space-time $M^4 \times T^2/Z_6 \times S^1/Z_2$ with coordinates x^μ , ($\mu = 0, 1, 2, 3$), x^5 , x^6 , and x^7 . The torus T^2 is homeomorphic to $S^1 \times S^1$ and the radii of the circles along the x^5 , x^6 , and x^7 directions are R_1 , R_2 , and R' , respectively. We introduce a complex coordinate z for T^2 and a real coordinate y for S^1 ,

$$z \equiv \frac{1}{2}(x^5 + ix^6), \quad y \equiv x^7. \quad (\text{B1})$$

The orbifold T^2/Z_6 has been defined in Appendix A, while the orbifold S^1/Z_2 is obtained from S^1 by moduloing the equivalent class

$$\Gamma_S: y \sim -y. \quad (\text{B2})$$

There are two fixed points: $y = 0$ and $y = \pi R'$.

The 7D $\mathcal{N} = 1$ supersymmetry has 16 supercharges corresponding to $\mathcal{N} = 4$ supersymmetry in 4D, and only the gauge multiplet can be introduced in the bulk. This

multiplet can be decomposed under 4D $\mathcal{N} = 1$ supersymmetry into a gauge vector multiplet V and three chiral multiplets Σ_1 , Σ_2 , and Σ_3 , all in the adjoint representation, where the fifth and sixth components of the gauge field, A_5 and A_6 , are contained in the lowest component of Σ_1 , and the seventh component of the gauge field A_7 is contained in the lowest component of Σ_2 .

We express the bulk action in the Wess-Zumino gauge and 4D $\mathcal{N} = 1$ supersymmetry notation [24]

$$\begin{aligned} S = \int d^7x & \left\{ \text{Tr} \left[\int d^2\theta \left(\frac{1}{4\kappa g^2} \mathcal{W}^\alpha \mathcal{W}_\alpha + \frac{1}{\kappa g^2} \left(\Sigma_3 \partial_z \Sigma_2 \right. \right. \right. \right. \\ & \left. \left. \left. + \Sigma_1 \partial_y \Sigma_3 - \frac{1}{\sqrt{2}} \Sigma_1 [\Sigma_2, \Sigma_3] \right) \right) + \text{H.c.} \right] \\ & + \int d^4\theta \frac{1}{\kappa g^2} \text{Tr} [(\sqrt{2} \partial_z^\dagger + \Sigma_1^\dagger) e^{-V} (-\sqrt{2} \partial_z + \Sigma_1) e^V \\ & + \partial_z^\dagger e^{-V} \partial_z e^V + (\sqrt{2} \partial_y + \Sigma_2^\dagger) e^{-V} (-\sqrt{2} \partial_y + \Sigma_2) e^V \\ & \left. + \partial_y e^{-V} \partial_y e^V + \Sigma_3^\dagger e^{-V} \Sigma_3 e^V \right] \}. \quad (\text{B3}) \end{aligned}$$

From the above action, we obtain the transformations of the vector multiplet:

$$V(x^\mu, \omega z, \omega^{-1} \bar{z}, y) = R_{\Gamma_T} V(x^\mu, z, \bar{z}, y) R_{\Gamma_T}^{-1}, \quad (\text{B4})$$

$$\Sigma_1(x^\mu, \omega z, \omega^{-1} \bar{z}, y) = \omega^{-1} R_{\Gamma_T} \Sigma_1(x^\mu, z, \bar{z}, y) R_{\Gamma_T}^{-1}, \quad (\text{B5})$$

$$\Sigma_2(x^\mu, \omega z, \omega^{-1} \bar{z}, y) = R_{\Gamma_T} \Sigma_2(x^\mu, z, \bar{z}, y) R_{\Gamma_T}^{-1}, \quad (\text{B6})$$

$$\Sigma_3(x^\mu, \omega z, \omega^{-1} \bar{z}, y) = \omega R_{\Gamma_T} \Sigma_3(x^\mu, z, \bar{z}, y) R_{\Gamma_T}^{-1}, \quad (\text{B7})$$

$$V(x^\mu, z, \bar{z}, -y) = R_{\Gamma_S} V(x^\mu, z, \bar{z}, y) R_{\Gamma_S}^{-1}, \quad (\text{B8})$$

$$\Sigma_1(x^\mu, z, \bar{z}, -y) = R_{\Gamma_S} \Sigma_1(x^\mu, z, \bar{z}, y) R_{\Gamma_S}^{-1}, \quad (\text{B9})$$

$$\Sigma_2(x^\mu, z, \bar{z}, -y) = -R_{\Gamma_S} \Sigma_2(x^\mu, z, \bar{z}, y) R_{\Gamma_S}^{-1}, \quad (\text{B10})$$

$$\Sigma_3(x^\mu, z, \bar{z}, -y) = -R_{\Gamma_S} \Sigma_3(x^\mu, z, \bar{z}, y) R_{\Gamma_S}^{-1}, \quad (\text{B11})$$

where we introduce nontrivial transformations R_{Γ_T} and R_{Γ_S} to break the bulk gauge group G .

The $Z_6 \times Z_2$ transformation properties for the decomposed components of V , Σ_1 , Σ_2 , and Σ_3 in the $SU(8)$ model are given by

$$V: \begin{pmatrix} (1, +) & (\omega^{-n_1}, +) & (\omega^{-n_1}, -) & (1, -) & (\omega^{-n_2}, +) \\ (\omega^{n_1}, +) & (1, +) & (1, -) & (\omega^{n_1}, -) & (\omega^{n_1-n_2}, +) \\ (\omega^{n_1}, -) & (1, -) & (1, +) & (\omega^{n_1}, +) & (\omega^{n_1-n_2}, -) \\ (1, -) & (\omega^{-n_1}, -) & (\omega^{-n_1}, +) & (1, +) & (\omega^{-n_2}, -) \\ (\omega^{n_2}, +) & (\omega^{n_2-n_1}, +) & (\omega^{n_2-n_1}, -) & (\omega^{n_2}, -) & (1, +) \end{pmatrix} + (1, +), \quad (\text{B12})$$

$$\Sigma_1: \left(\begin{array}{ccccc} (\omega^{-1}, +) & (\omega^{-n_1-1}, +) & (\omega^{-n_1-1}, -) & (\omega^{-1}, -) & (\omega^{-n_2-1}, +) \\ (\omega^{n_1-1}, +) & (\omega^{-1}, +) & (\omega^{-1}, -) & (\omega^{n_1-1}, -) & (\omega^{n_1-n_2-1}, +) \\ (\omega^{n_1-1}, -) & (\omega^{-1}, -) & (\omega^{-1}, +) & (\omega^{n_1-1}, +) & (\omega^{n_1-n_2-1}, -) \\ (\omega^{-1}, -) & (\omega^{-n_1-1}, -) & (\omega^{-n_1-1}, +) & (\omega^{-1}, +) & (\omega^{-n_2-1}, -) \\ (\omega^{n_2-1}, +) & (\omega^{n_2-n_1-1}, +) & (\omega^{n_2-n_1-1}, -) & (\omega^{n_2-1}, -) & (\omega^{-1}, +) \end{array} \right) + (\omega^{-1}, +), \quad (\text{B13})$$

$$\Sigma_2: \left(\begin{array}{ccccc} (1, -) & (\omega^{-n_1}, -) & (\omega^{-n_1}, +) & (1, +) & (\omega^{-n_2}, -) \\ (\omega^{n_1}, -) & (1, -) & (1, +) & (\omega^{n_1}, +) & (\omega^{n_1-n_2}, -) \\ (\omega^{n_1}, +) & (1, +) & (1, -) & (\omega^{n_1}, -) & (\omega^{n_1-n_2}, +) \\ (1, +) & (\omega^{-n_1}, +) & (\omega^{-n_1}, -) & (1, -) & (\omega^{-n_2}, +) \\ (\omega^{n_2}, -) & (\omega^{n_2-n_1}, -) & (\omega^{n_2-n_1}, +) & (\omega^{n_2}, +) & (1, -) \end{array} \right) + (1, -), \quad (\text{B14})$$

$$\Sigma_3: \left(\begin{array}{ccccc} (\omega, -) & (\omega^{-n_1+1}, -) & (\omega^{-n_1+1}, +) & (\omega, +) & (\omega^{-n_2+1}, -) \\ (\omega^{n_1+1}, -) & (\omega, -) & (\omega, +) & (\omega^{n_1+1}, +) & (\omega^{n_1-n_2+1}, -) \\ (\omega^{n_1+1}, +) & (\omega, +) & (\omega, -) & (\omega^{n_1+1}, -) & (\omega^{n_1-n_2+1}, +) \\ (\omega, +) & (\omega^{-n_1+1}, +) & (\omega^{-n_1+1}, -) & (\omega, -) & (\omega^{-n_2+1}, +) \\ (\omega^{n_2+1}, -) & (\omega^{n_2-n_1+1}, -) & (\omega^{n_2-n_1+1}, +) & (\omega^{n_2+1}, +) & (\omega, -) \end{array} \right) + (\omega, -). \quad (\text{B15})$$

From Eqs. (B12)–(B15), we obtain that the 7D $\mathcal{N} = 1$ supersymmetric $SU(8)$ gauge symmetry is broken down to 4D $\mathcal{N} = 1$ supersymmetric $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_\alpha \times U(1)_\beta \times U(1)_\gamma$ gauge symmetry [2].

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