Inflation with realistic supersymmetric $SO(10)$

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(Received 3 May 2005; revised manuscript received 8 August 2005; published 30 September 2005)

We implement inflation within a realistic supersymmetric $SO(10)$ model in which the doublet-triplet splitting is realized through the Dimopoulos-Wilczek mechanism, the MSSM μ problem is resolved, and Higgsino mediated dimension five nucleon decay is heavily suppressed. The cosmologically unwanted topological defects are inflated away, and from $\delta T/T$, the $B-L$ breaking scale is estimated to be of order 10^{16}–10^{17} GeV. Including supergravity corrections, the scalar spectral index $n_s = 0.99 \pm 0.01$, with $|dn_s/d\ln k| \lesssim 10^{-3}$.

DOI: 10.1103/PhysRevD.72.063515 PACS numbers: 98.80.Cq, 12.10.Dm

In an attractive class of supersymmetric (SUSY) models, inflation is associated with spontaneous breaking of a gauge symmetry, such that $\delta T/T$ is proportional to $(M/M_{\rm Planck})^2$, where M denotes the symmetry breaking scale and $M_{\rm Planck}$ ($\equiv 1.2 \times 10^{19}$ GeV) denotes the Planck mass [1,2]. Thus, from measurements of $\delta T/T$, M is estimated to be of order 10^{16} GeV [1,3,4]. The scalar spectral index n_s in these models is very close to unity,[1] in excellent agreement with recent fits to the data [5].[2] A $U(1)$ R symmetry plays an essential role in the construction of these inflationary models. These models possess another important property, namely, with the minimal Kähler potential, the supergravity (SUGRA) corrections do not spoil the inflationary scenario [2,8], which has been realized with a variety of attractive gauge groups including $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ ($\equiv G_{LR}$) [9], $SU(4)_c \times SU(2)_L \times SU(2)_R$ ($\equiv G_{422}$) [10], and $SU(5)$ [11]. (The gauge symmetries G_{LR} and G_{422} were first introduced in Refs. [12,13].) Our goal in this paper is to implement inflation in a realistic $SO(10)$ model.

$SO(10)$ [14] has two attractive features that it shares with G_{422}, namely, it predicts the existence of right-handed neutrinos as well as the seesaw mechanism. These two features are very helpful in understanding neutrino oscillations [15] and also in generating a baryon asymmetry via leptogenesis [16]. Furthermore, at least within a four dimensional setting, it seems easier to realize doublet-triplet (DT) splitting without fine tuning in $SO(10)$ (say via the Dimopoulos-Wilczek mechanism [17]) than in $SU(5)$.

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[1]Following Ref. [4], including supergravity corrections, these models can yield a spectral index somewhat larger than unity. It is in our current model that $n_s = 0.99 \pm 0.01$, as we emphasize in the abstract.

[2]Earlier there was some (weak) evidence for a running spectral index with $dn_s/d\ln k \approx -5 \times 10^{-2}$. But this is not confirmed by a more recent analysis [6]. We will consider the simplest models which yield $dn_s/d\ln k \lesssim -10^{-3}$. However, more complicated scenarios with two or more inflationary epochs can yield a significantly larger $dn_s/d\ln k$ [7].

To implement $SO(10)$ inflation we would like to work with a realistic model with the following properties: DT splitting is realized without fine tuning, and the low energy theory coincides with the minimal supersymmetric standard model (MSSM). [For $SO(10)$ inflation in a five dimensional setting, see Ref. [18].] The MSSM μ problem should also be resolved, and Higgsino mediated dimension five nucleon decay should be adequately suppressed. Gauge boson mediated nucleon decay is still present with a predicted nucleon lifetime of order 10^{34}–10^{36} yrs. Finally, matter parity is unbroken, so that the lightest supersymmetric particle (LSP) is stable and makes up the dark matter in the Universe.

To achieve natural DT splitting and the MSSM at low energies with $SO(10)$, one is led to consider a nonminimal set of Higgs superfields. This is to be contrasted with the subgroups of $SO(10)$, such as G_{LR} or G_{422} above, in which the DT splitting problem is absent. Many authors have previously addressed the DT splitting and the dimension five nucleon decay problem in $SO(10)$ [19–21], and the proposed solutions are not necessarily straightforward. In this paper we will follow Refs. [20,21], with suitable modifications needed to make the scheme consistent with the desired inflationary scenario, and also to avoid potential cosmological problems (monopoles, moduli, etc.). While doing this we would like to also ensure that the SUGRA corrections do not disrupt the inflationary scenario.

A minimal set of Higgs required to break $SO(10)$ to the MSSM gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$ ($\equiv G_{\rm SM}$) is $\mathbf{45}_H$, $\mathbf{16}_H$, $\overline{\mathbf{16}}_H$. A nonzero vacuum expectation value (VEV) of $\mathbf{45}_H$ along the $B-L$ (I_{3R}) direction breaks $SO(10)$ to $G_{\rm LR}$ ($SU(4)_c \times SU(2)_L \times U(1)_R$) and produces magnetic monopoles. The $\mathbf{16}_H$, $\overline{\mathbf{16}}_H$ VEVs break $SO(10)$ to $SU(5)$ and induce masses for the right-handed neutrinos via dimension five operators. [Note that breaking of $SO(10)$ with $\mathbf{16}_H + \overline{\mathbf{16}}_H$, in contrast to $\mathbf{126}_H + \overline{\mathbf{126}}_H$, does not produce Z_2 cosmic strings [22].] One of our goals, of course, is to make sure that the topological defects do not pose cosmological difficulties. Thus,

TABLE I. $U(1)$ R and $U(1)_A$ charge assignments for the superfields.

	S	X	X'	Y	P	$\overline{P}$	Q	$\overline{Q}$	$\mathbf{10}$	$\mathbf{10}_h$
R	1	−1	−1	0	0	0	−2	2	1	0
A	0	−2/3	−2/3	−1/3	−1/4	1/4	−1/2	1/2	1/6	0
	$\mathbf{16}$	$\overline{\mathbf{16}}$	$\mathbf{16}'$	$\overline{\mathbf{16}}'$	$\mathbf{16}_H$	$\overline{\mathbf{16}}_H$	$\mathbf{16}_3$	$\mathbf{45}$	$\mathbf{45}_H$	$\mathbf{45}'_H$
R	1	3	2	2	0	0	1/2	1	0	−1
A	1/2	2/3	2/3	2/3	0	0	0	1/2	−1/6	−1/3

it would be helpful if during inflation $SO(10)$ is, for instance, broken to G_{LR} [23], $SU(4)_c \times SU(2)_L \times U(1)_R$, or $G_{\rm SM}$.

To implement DT splitting without fine tuning and eliminate dimension five proton decay, and to recover the MSSM at low energies with the μ problem resolved, we need an additional **45**-plet ($\mathbf{45}'_H$), two additional $\mathbf{16} + \overline{\mathbf{16}}$ pairs, two **10**-plets ($\mathbf{10}_h$ and $\mathbf{10}$), and several singlets [20,21]. One more **45**-plet is also required by $U(1)$ R symmetry. This symmetry, among other things, plays an essential role in realizing inflation, and its Z_2 subgroup coincides with the MSSM matter parity. The $SO(10)$ singlet superfields are denoted as S, X, X', Y, P, $\overline{P}$, Q, and $\overline{Q}$, whose roles will be described below. Table I displays the quantum numbers (under the global $U(1)$ R and $U(1)_A$ symmetries) of all the Higgs sector superfields and the third family matter field ($\mathbf{16}_3$). Following standard practice, we employ the same notation for the superfields and their scalar components.

To break $SO(10)$ to G_{LR}, consider the superpotential,

$$W_{45} = \frac{\alpha}{6M_*} X^{(\prime)} Y \,\mathrm{Tr}(\mathbf{4545}) - \frac{\beta}{6} Y\,\mathrm{Tr}(\mathbf{4545}_H) + \frac{\gamma_1}{36M_*}\mathrm{Tr}(\mathbf{4545}_H)\mathrm{Tr}(\mathbf{45}_H\mathbf{45}_H) + \frac{\gamma_2}{6M_*}\mathrm{Tr}(\mathbf{4545}_H\mathbf{45}_H\mathbf{45}_H), \quad (1)$$

where α, β, $\gamma_{1,2}$ are dimensionless parameters, and M_* ($\sim 10^{18}$ GeV) denotes the cutoff scale. As will be explained, X, X', and Y can develop nonzero VEVs, $\langle X\rangle \sim \langle X'\rangle \sim \langle Y\rangle \sim 10^{16}$ GeV. Because of nonzero $\langle Y\rangle$, $\mathbf{45}_H$ can also obtain a VEV in the $B-L$ direction from the β and $\gamma_{1,2}$ terms of Eq. (1),

$$\langle \mathbf{45}_H\rangle = \begin{pmatrix} v & & & & \\ & v & & & \\ & & v & & \\ & & & 0 & \\ & & & & 0 \end{pmatrix} \otimes i\sigma_2, \quad \text{and} \quad \langle \mathbf{45}\rangle = 0, \quad (2)$$

where $v = \sqrt{\frac{\beta}{\gamma}\langle Y\rangle M_*} \equiv M_{\rm GUT}$ ($\approx 3\times 10^{16}$ GeV), with $\gamma \equiv \gamma_1 + \gamma_2$. The 3×3 block corresponds to $SU(3)_c$ and the 2×2 block to $SU(2)_L$ of the MSSM gauge group. Hence, the $SO(10)$ gauge symmetry is broken to G_{LR}. Note that from the "α term," the **45** multiplet becomes superheavy. It acquires a VEV of order $(m_{3/2}M_{\rm GUT})/M_*$ after SUSY breaking, where $m_{3/2}$ ($\sim$ TeV) denotes the scale of the soft parameters.

The next step in the breaking to the MSSM gauge group $G_{\rm SM}$ ($= G_{LR} \cap SU(5)$) is achieved with the following superpotential,

$$\begin{aligned} W_{16} = {}& S[\kappa \mathbf{16}_H\overline{\mathbf{16}}_H + \lambda \mathbf{10}_h\mathbf{10}_h - \kappa M_{B-L}^2] \\ & - \frac{\rho}{M_*^2} S(\mathbf{16}_H\overline{\mathbf{16}}_H)^2 + \mathbf{16}\left[\frac{\lambda_1}{M_*}\mathbf{45}_H Y - \frac{\lambda_2}{M_*}P^2\right]\overline{\mathbf{16}}_H \\ & + \overline{\mathbf{16}}\left[\frac{\lambda_3}{M_*}\mathbf{45}_H Q - \frac{\lambda_4}{M_*}(\mathbf{45}'_H)^2\right]\mathbf{16}_H \\ & + \mathbf{16}'\left[\frac{\lambda_5}{M_*}\mathbf{45}'_H Y - \lambda_6 X\right]\overline{\mathbf{16}}_H \\ & + \overline{\mathbf{16}}'\left[\frac{\lambda_7}{M_*}\mathbf{45}'_H Y - \lambda_8 X'\right]\mathbf{16}_H, \end{aligned} \quad (3)$$

where ρ is a dimensionless coupling constant. The dimensionful parameter M_{B-L}, as determined from inflation ($\delta T/T$), turns out to be of order 10^{16}–10^{17} GeV [4]. The superfield $\mathbf{10}_h$ includes the two MSSM Higgs doublets. As previously mentioned, additional $\mathbf{16}$, $\overline{\mathbf{16}}$ are essential to stabilize the VEV of $\mathbf{45}_H$ in Eq. (2) [20]. From the κ and ρ terms, $\mathbf{16}_H$ and $\overline{\mathbf{16}}_H$ develop VEVs of order M_{B-L}, breaking $SO(10)$ to $SU(5)$,

$$|\langle \mathbf{16}_H\rangle|^2 = |\langle \overline{\mathbf{16}}_H\rangle|^2 = \frac{M_{B-L}^2}{2\zeta}\left[1 - \sqrt{1-4\zeta}\right], \quad (4)$$

where $\zeta \equiv \rho M_{B-L}^2/(\kappa M_*^2)$ [10], while $\langle S\rangle = \langle \mathbf{10}_h\rangle = 0$ up to corrections of $O(m_{3/2})$ by including soft SUSY breaking terms in the scalar potential [9]. The "D-term" scalar potential vanishes along the (D-flat) direction $|\langle \mathbf{16}_H\rangle| = |\langle \overline{\mathbf{16}}_H^*\rangle|$ ($= |\langle \overline{\mathbf{16}}_H\rangle|$). Together with Eq. (2), the $SO(10)$ gauge symmetry is broken to the MSSM gauge symmetry. The MSSM Higgs doublets arise from $\mathbf{10}_h$. With $\langle S\rangle \approx -m_{3/2}/\kappa$, the μ term from Eq. (3) is of order $(\lambda/\kappa)m_{3/2} \sim$ TeV, for $\kappa \approx \lambda$.[3] Similarly the soft term $B\mu$ ($\approx -2(\lambda/\kappa)m_{3/2}^2$) is generated [9,24].

[3]From the nonrenormalizable term $y_\mu \mathbf{1010}_h\langle \mathbf{16}_H\mathbf{45}_H\overline{\mathbf{16}}_H\rangle/M_*^2$, the doublets in $\mathbf{10}_h$ obtains a "seesaw mass" $y_\mu^2(\langle \mathbf{16}_H\mathbf{45}_H\overline{\mathbf{16}}_H\rangle)^2/(M_*^4\langle \mathbf{45}'_H\rangle) \sim$ TeV with $y_\mu \sim 10^{-3}$, which modifies the μ parameter at low energies.

Our next step is to ensure that the low energy theory coincides precisely with the MSSM. With $SO(10)$ broken to G_{LR} by $\langle \mathbf{45}_H \rangle$ via Eq. (1), the Goldstone modes from $\mathbf{45}_H$, $[\{(\mathbf{3}, \overline{\mathbf{2}})_{-5/6}, (\mathbf{3}, \mathbf{2})_{1/6}, (\overline{\mathbf{3}}, \mathbf{1})_{-2/3}\} + \text{H.c.}]$ in terms of $G_{\rm SM}$, are absorbed by the gauge sector. The states of $(\mathbf{8}, \mathbf{1})_0$, $(\mathbf{1}, \mathbf{3})_0$, $(\mathbf{1}, \mathbf{1})_0$, $(\mathbf{1}, \mathbf{1})_1$, and $(\mathbf{1}, \mathbf{1})_{-1}$ contained in $\mathbf{45}_H$ acquire superheavy masses through the quartic couplings. On the other hand, when $SO(10)$ breaks to $SU(5)$ by $\langle \mathbf{16}_H \rangle$ and $\langle \overline{\mathbf{16}}_H \rangle$, the states $[\{(\mathbf{3}, \mathbf{2})_{1/6}, (\overline{\mathbf{3}}, \mathbf{1})_{-2/3}, (\mathbf{1}, \mathbf{1})_1\} + \text{H.c.}] + (\mathbf{1}, \mathbf{1})_0$ in $\mathbf{16}_H$, $\overline{\mathbf{16}}_H$ should be absorbed by the gauge sector, while $[\{(\overline{\mathbf{3}}, \mathbf{1})_{1/3}, (\mathbf{1}, \overline{\mathbf{2}})_{-1/2}\} + \text{H.c.}]$ remain massless (or light). Note that $[\{(\mathbf{3}, \mathbf{2})_{1/6}, (\overline{\mathbf{3}}, \mathbf{1})_{-2/3}\} + \text{H.c.}]$ are common between $\mathbf{45}_H$ and $\mathbf{16}_H$, $\overline{\mathbf{16}}_H$. Thus, when $SO(10)$ breaks to $G_{\rm SM}$ by an adjoint and a vectorlike pair of spinorial Higgs, the superfields associated with $[\{(\mathbf{3}, \mathbf{2})_{1/6}, (\overline{\mathbf{3}}, \mathbf{1})_{-2/3}, (\overline{\mathbf{3}}, \mathbf{1})_{1/3}, (\mathbf{1}, \overline{\mathbf{2}})_{-1/2}\} + \text{H.c.}]$ are pseudo-Goldstone modes. The extra light multiplets would spoil the unification of the MSSM gauge couplings, and therefore must be eliminated.

The simplest way to remove them from the low energy spectrum is to introduce couplings such as $\mathbf{16}_H\mathbf{45}_H\overline{\mathbf{16}}_H$. However, the presence of such a term in the superpotential destabilizes the form of $\langle \mathbf{45}_H \rangle$ given in Eq. (2), in such a way that at the SUSY minimum, $v = 0$ is required. It was shown in Ref. [20] that with the "λ_i" couplings ($i = 1, 2, 3, 4$) and an additional $\mathbf{16}$–$\overline{\mathbf{16}}$ pair in Eq. (3), the unwanted pseudo-Goldstone modes all become superheavy, keeping intact the form of Eq. (2) at the SUSY minimum.

From the "F-flat conditions" with $\mathbf{16}_H$ and $\overline{\mathbf{16}}_H$ acquiring nonzero VEVs, one finds

$$\langle \mathbf{45}_H \rangle \langle Y \rangle = \frac{\lambda_2}{\lambda_1} \langle P^2 \rangle, \qquad \langle \mathbf{45}_H \rangle \langle Q \rangle = \frac{\lambda_4}{\lambda_3} \text{Tr} \langle \mathbf{45}'_H \rangle^2. \quad (5)$$

Thus, if P and Q develop VEVs, $\langle \mathbf{45}_H \rangle$, $\langle \mathbf{45}'_H \rangle$, and $\langle Y \rangle$ should also appear. We will soon explain how $\langle P \rangle$ and $\langle Q \rangle$ arise. Since $\langle Y \rangle$ is related to $\langle \mathbf{45}_H \rangle$ via Eq. (2), both are uniquely determined. We assume that $\langle \mathbf{45}'_H \rangle$ points in the I_{3R} direction,

$$\langle \mathbf{45}'_H \rangle = \begin{pmatrix} 0 & & & & \\ & 0 & & & \\ & & 0 & & \\ & & & v' & \\ & & & & v' \end{pmatrix} \otimes i\sigma_2. \quad (6)$$

Recall that $\langle \mathbf{45}'_H \rangle$ is employed to suppress Higgsino mediated dimension five nucleon decay [21]. Similarly, due to the presence of the "λ_i" ($i = 5, 6, 7, 8$) couplings in Eq. (3), the low energy spectrum is protected even with the $\mathbf{45}'_H$ present [21]. With nonzero VEVs for $\mathbf{45}'_H$ and Y, X and X' slide to values satisfying

$$\frac{\lambda_{5,7}}{M_*} \langle \mathbf{45}'_H \rangle \langle Y \rangle - \lambda_{6,8} \langle X^{(\prime)} \rangle = 0, \quad (7)$$

with $|\langle \mathbf{16}' \rangle| = |\langle \overline{\mathbf{16}} \rangle| \sim O(m_{3/2})$. In order to guarantee the "λ_i" couplings in Eq. (3) and to forbid $\mathbf{16}_H\mathbf{45}_H\overline{\mathbf{16}}_H$, the $U(1)$ symmetries in Table I are essential.

To obtain nonvanishing VEVs for P and Q, one could, as a simple example, consider the following superpotential,

$$W_{PQ} = S[\kappa_1 P\overline{P} + \kappa_2 Q\overline{Q}] - \frac{S}{M_*^2}[\rho_1 (P\overline{P})^2 + \rho_2 (Q\overline{Q})^2], \quad (8)$$

such that

$$\langle P\overline{P} \rangle = \frac{\kappa_1}{\rho_1} M_*^2 \sim M_{\rm GUT}^2, \quad \langle Q\overline{Q} \rangle = \frac{\kappa_2}{\rho_2} M_*^2 \sim M_{\rm GUT}^2. \quad (9)$$

The $\lambda_{2,3}$ terms in Eq. (3) just determine $\langle \mathbf{45}_H \rangle$, $\langle Y \rangle$, and $\langle \mathbf{45}'_H \rangle$. With the inclusion of soft SUSY breaking terms, the VEVs $\langle P \rangle$, $\langle \overline{P} \rangle$, $\langle Q \rangle$, and $\langle \overline{Q} \rangle$ would be completely fixed. To avoid potential cosmological problems associated with moduli fields, we make the important assumption that the VEVs satisfy the constraints $\langle P \rangle = \langle \overline{P} \rangle$ and $\langle Q \rangle = \langle \overline{Q} \rangle$. This could be made plausible by assuming universal soft scalar masses, and that the SUSY breaking "A terms" asymmetric under $P \leftrightarrow \overline{P}$ and $Q \leftrightarrow \overline{Q}$ are plausibly small enough.[4,5] Note that even with the soft SUSY breaking terms in the Lagrangian, the grand unified theory (GUT) scale results Eqs. (5) and (9) should still be effectively valid. Since the fields that couple to P, $\overline{P}$, Q, and $\overline{Q}$ are all superheavy, the soft parameters are expected to be radiatively stable at low energies. Thus, at the minimum of the scalar potential, we have four mass eigenstates, $(P \pm \overline{P})/\sqrt{2}$ ($\equiv P_\pm$) and $(Q \pm \overline{Q})/\sqrt{2}$ ($\equiv Q_\pm$). While P_+ and Q_+ obtain superheavy masses of order $M_{\rm GUT}$ and large VEVs ($= \sqrt{\kappa_{1,2}/\rho_{1,2}} M_* + O(m_{3/2}) \sim M_{\rm GUT}$, respectively), P_- and Q_- remain light ($\sim m_{3/2}$) with vanishing VEVs.

With $\langle \mathbf{45}_H \rangle$ in Eq. (2), the "DT splitting problem" resolves itself through the mechanism in [17]. Consider the superpotential

$$W_{10} = y_1 \mathbf{10}\mathbf{45}'_H\mathbf{10} + y_2 \mathbf{10}\mathbf{45}_H\mathbf{10}_h. \quad (10)$$

From the first term in Eq. (10), only the doublets contained in $\mathbf{10}$ become superheavy [21], and from the second term

[4]In the gravity mediated SUSY breaking scenario with the minimal Kähler potential, A terms are given by $m_{3/2} \times [(A - 3)W + \sum_i \phi_i \frac{\partial W}{\partial \phi_i} + \text{H.c.}]$, where A is a dimensionless number associated with hidden sector dynamics [25]. Since dimensions of the operators associated with the λ_k's ($k = 1, 2, 3, 4$) in Eq. (3) are all the same, the A-term coefficients ($\equiv A_{\lambda_k}$) corresponding to λ_k should be $m_{3/2}(A + 1)\lambda_k$, and so satisfy $A_{\lambda_{j+1}}/A_{\lambda_j} = \lambda_{j+1}/\lambda_j$ ($j = 1, 3$). Hence, at the minimum, the A terms corresponding to λ_k are canceled by each other with the VEVs in Eq. (5). Since the other soft terms are symmetric under $P \leftrightarrow \overline{P}$ and $Q \leftrightarrow \overline{Q}$, we have $\langle P \rangle = \langle \overline{P} \rangle$ and $\langle Q \rangle = \langle \overline{Q} \rangle$ at the minimum of the scalar potential.

[5]In gauge mediated SUSY breaking scenario, A terms are generally suppressed.

only the color triplet fields included in **10** and $\mathbf{10}_h$ acquire superheavy masses [17,20,21]. Since the two color triplets contained in $\mathbf{10}_h$ do not couple in Eq. (10), dimension five nucleon decay which may be in conflict with the Superkamiokande observations [15] is eliminated in the SUSY limit [21]. Note that operators such as $\mathbf{1010}_h$, $\mathbf{10}_h\mathbf{10}_h$, $[\mathbf{1010}_h]\mathrm{Tr}(\mathbf{45}_H\mathbf{45}_H)$ and so on are allowed by $SO(10)$ and, unless forbidden, would destroy the gauge hierarchy. The $U(1)$ symmetries in Table I are once again crucial in achieving this.

Although the superpotential coupling $\langle S\rangle\mathbf{10}_h\mathbf{10}_h$ induces Higgsino mediated dimension five nucleon decay, there is a huge suppression factor of $m_{3/2}/M_{\rm GUT}$. Thus, we expect that nucleon decay is dominated by the exchange of the superheavy gauge bosons with an estimated lifetime $\tau_p \to e^+\pi^0$ of order 10^{34}–10^{36} yrs. Note that we have assumed that dimension five operators such as $\mathbf{16}_i\mathbf{16}_j\mathbf{16}_k\mathbf{16}_l$, $\mathbf{16}_i\mathbf{16}_j\mathbf{16}_k\mathbf{16}_H$, and so on, where the subscripts are family indices of the matter, are adequately suppressed by assigning suitable R and A charges to these matter superfields. This is closely tied to the flavor problem, which we will not address here.

Consider next the superpotential couplings involving the third generation matter superfields,

$$W_m = y_3\mathbf{16}_3\mathbf{16}_3\mathbf{10}_h + \frac{y_\nu}{M_*}\mathbf{16}_3\mathbf{16}_3\overline{\mathbf{16}}_H\overline{\mathbf{16}}_H. \quad (11)$$

The first term yields Yukawa unification so that the MSSM parameter $\tan\beta \approx m_t/m_b$ [26]. For a realistic construction of the fermion's mass matrices in $SO(10)$, refer to e.g. Ref. [27]. From the y_ν term, the right-handed neutrino masses are $\lesssim y_\nu M_{B-L}^2/M_* \sim 10^{14}$ GeV. Right-handed neutrino masses of order 10^{14} GeV and smaller can yield a mass spectrum for the light neutrinos through the seesaw mechanism that is suitable for neutrino oscillations. These masses are also appropriate for realizing leptogenesis after inflation [7,28]. Finally let us note that the $\mathbf{16}_H$, $\overline{\mathbf{16}}_H$ VEVs break the center Z_4 of $SO(10)$ completely [22]. The role of "matter parity" is played by the unbroken Z_2 subgroup of the $U(1)$ R symmetry [9]. Thus the LSP in our model is expected to be stable and contribute to the dark matter in the Universe.

For completeness, we need to present also the other possible terms in the superpotential that were not discussed in Eqs. (1), (3), (8), (10), and (11). Indeed, we have more quartic couplings: $\mathbf{104510}_hX$, $\mathbf{16}\overline{\mathbf{16}}XQ$, $\mathbf{1616}_H\mathbf{10}X$, $\mathbf{16}'\mathbf{16}_H\mathbf{10}_hX$, $\mathbf{16}_H\mathbf{16}_H\mathbf{10}_hS$, $\mathbf{16}_H\mathbf{16}_H\mathbf{1045}_H$, $\overline{\mathbf{16}}'\overline{\mathbf{16}}_H\mathbf{10}_hX$, $\overline{\mathbf{16}}_H\overline{\mathbf{16}}_H\mathbf{10}_hS$, $\overline{\mathbf{16}}_H\overline{\mathbf{16}}_H\mathbf{1045}_H$, and so on, which also are consistent with the charge assignment in Table I. But they just provide subdominant effects in this model. For instance, $\mathbf{16}\overline{\mathbf{16}}\langle XQ\rangle$ cannot change the symmetry breaking pattern discussed above, because $\langle\mathbf{16}\rangle = \langle\overline{\mathbf{16}}\rangle = 0$. Thus, with keeping massless Goldstones, it just modifies the masses of the pseudo-Goldstone modes contained in **16** and $\overline{\mathbf{16}}$. Because of $\langle\mathbf{16}_H\rangle \neq 0$ and $\langle\mathbf{45}_H\rangle \neq 0$, $\mathbf{16}_H\mathbf{16}_H\mathbf{1045}_H$ slightly changes masses of the $SU(2)_L$ doublets contained in $\mathbf{16}_H$ and **10**. We also have over 40 extra penta-couplings except those considered in Eqs. (3) and (8), but we will neglect them.

Let us now discuss how inflation is implemented in the model described so far. In particular, we aim to show that the SUGRA corrections do not significantly affect the inflationary scenario, which is a nontrivial result in inflationary model building. The F-term scalar potential in SUGRA is given by

$$V_F = e^{K/M_P^2}\left[\sum_{i,j}(K^{-1})^i_j(D_{\phi_i}W)(D_{\phi_j}W)^* - 3\frac{|W|^2}{M_P^2}\right], \quad (12)$$

where M_P ($\equiv M_{\rm Planck}/\sqrt{8\pi} = 2.4\times 10^{18}$ GeV) denotes the reduced Planck mass. K ($= K(\phi_i, \phi_j^*) = K^*$) and W ($= W(\phi_i)$) are the Kähler potential and the superpotential, respectively. $(K^{-1})^i_j$ in Eq. (12) denotes the inverse of $\partial^2K/\partial\phi_i\partial\phi_j^*$. In our case, W is composed of Eqs. (1), (3), (8), (10), and (11). $D_{\phi_i}W$ in Eq. (12) is defined as

$$D_{\phi_i}W \equiv \frac{\partial W}{\partial\phi_i} + \frac{\partial K}{\partial\phi_i}\frac{W}{M_P^2}. \quad (13)$$

The Kähler potential can be expanded as $K = |\phi_i|^2 + c_4|\phi_i|^4/M_P^2 + \cdots$. For simplicity, we consider the minimal case with $\partial^2K/\partial\phi_i\partial\phi_j^* = \delta^i_j$. Indeed, as explained in [2], higher order terms in K (with a coefficient $\lesssim 10^{-2}$ for the quartic term) do not seriously affect inflation. For simplicity, we will also ignore the TeV scale electroweak symmetry breaking effects when discussing inflation.

In this paper, we aim to employ the "shifted" hybrid inflationary scenario proposed in Ref. [10], in which symmetries can be broken during inflation unlike the simple "hybrid inflation" model [1]. An inflationary scenario is realized in the early Universe with the scalar fields S, $\mathbf{16}_H$, $\overline{\mathbf{16}}_H$, P, $\overline{P}$, Q, and $\overline{Q}$ displaced from the present values. We suppose that initially $|\langle S\rangle|^2 \gtrsim M_{B-L}^2[1/(4\zeta) - 1]/2$ with $1/4 < \zeta < 1/7.2$ [10], and $\langle\mathbf{16}_H\rangle$, $\langle\overline{\mathbf{16}}_H\rangle$, $\langle P\rangle$, $\langle\overline{P}\rangle$, $\langle Q\rangle$, $\langle\overline{Q}\rangle \neq 0$ with the inflationary superpotential given by [10]

$$W_{\rm infl} \approx -\kappa S\left[M_{B-L}^2 - \mathbf{16}_H\overline{\mathbf{16}}_H + \frac{\rho}{\kappa M_*^2}(\mathbf{16}_H\overline{\mathbf{16}}_H)^2 - \frac{\kappa_1}{\kappa}P\overline{P} + \frac{\rho_1}{\kappa M_*^2}(P\overline{P})^2 - \frac{\kappa_2}{\kappa}Q\overline{Q} + \frac{\rho_2}{\kappa M_*^2}(Q\overline{Q})^2\right] \quad (14)$$

$$\equiv -\kappa S M_{\rm eff}^2,$$

where $M_{\rm eff}^2$ turns out to be of order M_{B-L}^2. With $D_SW \approx -\kappa M_{\rm eff}^2(1 + |S|^2/M_P^2)$, one can see that the F-term scalar potential becomes

$$V_F \approx \left(1 + \sum_k \frac{|\phi_k|^2}{M_P^2} + \cdots\right)\left[\kappa^2 M_{\rm eff}^4\left(1 + \frac{|S|^4}{2M_P^4}\right) + \left(1 + \frac{|S|^2}{M_P^2} + \frac{|S|^4}{2M_P^4}\right)\sum_k |D_{\phi_k} W|^2\right], \quad (15)$$

where all scalar fields except S contribute to ϕ_k. The factor $(1 + \sum_k |\phi_k|^2/M_P^2 + \cdots)$ in front originates from e^{K/M_P^2} in Eq. (12). In Eq. (15) the quadratic term in S from $|D_S W|^2$, which is of order $(\kappa^2 M_{\rm eff}^4/M_P^2)|S|^2$ $(\approx H^2|S|^2)$, has canceled out with the factor "$-3|W|^2/M_P^2$" $(\approx -3\kappa^2 M_{\rm eff}^4 |S|^2/M_P^2)$ and the quadratic term in S from "e^{K/M_P^2}" $(= 1 + |S|^2/M_P^2 + \cdots)$. It is a common feature in this class of models [2]. Thus, only if $|D_{\phi_k} W|/M_P$'s are much smaller than the Hubble scale $(\sim \kappa M_{\rm eff}^2/M_P)$, the flatness of S will be guaranteed even with the SUGRA corrections included. Note that the $U(1)$ R symmetry ensures the absence of terms proportional to S^2, S^3, etc. in the superpotential, which otherwise could spoil the slow-roll conditions.

Let us consider the inflationary trajectory on which $\langle \mathbf{10}\rangle = \langle \mathbf{10}_h\rangle = \langle \mathbf{16}\rangle = \langle \overline{\mathbf{16}}\rangle = \langle \mathbf{16}'\rangle = \langle \overline{\mathbf{16}}'\rangle = \langle \mathbf{16}_3\rangle = \langle \mathbf{45}\rangle = 0$, with $D_{\mathbf{10}}W = D_{\mathbf{10}_h}W = D_{\mathbf{16}}W = D_{\overline{\mathbf{16}}}W = D_{\mathbf{16}'}W = D_{\overline{\mathbf{16}}'}W = D_{\mathbf{16}_3}W = D_{\mathbf{45}}W = 0$. On the other hand,

$$D_{\mathbf{16}_H} W = \kappa S\left[\overline{\mathbf{16}}_H\left(1 - \frac{2\rho}{\kappa M_*^2}(\mathbf{16}_H\overline{\mathbf{16}}_H)^2\right) - \mathbf{16}_H^* \frac{M_{\rm eff}^2}{M_P^2}\right], \quad (16)$$

$$D_P W = \kappa S\left[\overline{P}\left(\frac{\kappa_1}{\kappa} - \frac{2\rho_1}{\kappa M_*^2}(P\overline{P})^2\right) - P^* \frac{M_{\rm eff}^2}{M_P^2}\right], \quad (17)$$

$$D_Q W = \kappa S\left[\overline{Q}\left(\frac{\kappa_2}{\kappa} - \frac{2\rho_2}{\kappa M_*^2}(Q\overline{Q})^2\right) - Q^* \frac{M_{\rm eff}^2}{M_P^2}\right], \quad (18)$$

and similarly $D_{\overline{\mathbf{16}}_H}W = D_{\mathbf{16}_H}W(\mathbf{16}_H \leftrightarrow \overline{\mathbf{16}}_H)$, $D_{\overline{P}}W = D_P W(P \leftrightarrow \overline{P})$, and $D_{\overline{Q}}W = D_Q W(Q \leftrightarrow \overline{Q})$. The other $D_{\phi_l}W$'s ($\phi_l = X^{(\prime)}$, Y, $\mathbf{45}_H$, $\mathbf{45}_H'$) are approximately given by $-s\langle\phi_l^*\rangle$, where $s \equiv -W/M_P^2 \approx \kappa\langle S\rangle M_{\rm eff}^2/M_P^2$ $(<< M_{\rm GUT})$. At one of the local minima, $\langle \mathbf{16}_H\rangle$, $\langle \overline{\mathbf{16}}_H\rangle$, $\langle P\rangle$, $\langle \overline{P}\rangle$, $\langle Q\rangle$, $\langle \overline{Q}\rangle$, and the vacuum energy $V_0^{1/4}$ acquire the following values,

$$|\langle \mathbf{16}_H\rangle|^2 = |\langle \overline{\mathbf{16}}_H\rangle|^2 \approx \frac{\kappa M_*^2}{2\rho}\left[1 - \frac{M_{B-L}^2}{M_P^2} + \frac{\kappa M_*^2}{2\rho M_P^2} \times \left(1 - \frac{M_{B-L}^2}{4S^2} + O(\kappa_{1,2}^2/\kappa^2)\right)\right], \quad (19)$$

$$|\langle P\rangle|^2 = |\langle \overline{P}\rangle|^2 \approx \frac{\kappa_1 M_*^2}{2\rho_1}\left[1 - \frac{\kappa M_{B-L}^2}{\kappa_1 M_P^2} + \frac{\kappa^2 M_*^2}{4\kappa_1 \rho M_P^2} \times (1 + O(\kappa_{1,2}^2/\kappa^2))\right], \quad (20)$$

$$|\langle Q\rangle|^2 = |\langle \overline{Q}\rangle|^2 \approx \frac{\kappa_2 M_*^2}{2\rho_2}\left[1 - \frac{\kappa M_{B-L}^2}{\kappa_2 M_P^2} + \frac{\kappa^2 M_*^2}{4\kappa_2 \rho M_P^2} \times \left(1 + O(\kappa_{1,2}^2/\kappa^2)\right)\right], \quad (21)$$

$$V_0 \approx \kappa^2 M_0^4\left[1 + \frac{M_*^2}{M_P^2}\left(\frac{\kappa}{\rho} + \frac{\kappa_1}{\rho_1} + \frac{\kappa_2}{\rho_2} + O(\kappa^2, \kappa M_{B-L}^2/M_P^2)\right) + \sum_l \frac{|\langle\phi_l\rangle|^2}{M_P^2}\right], \quad (22)$$

where we assumed $\frac{\kappa}{\rho} \gtrsim \frac{\kappa_1}{\rho_1}, \frac{\kappa_2}{\rho_2}$ with $\kappa << 1$ and $\rho \sim \rho_1 \sim \rho_2 \sim O(1)$. In Eq. (22), $M_0^4 \equiv M_{B-L}^4[1/(4\zeta) + 1/(4\zeta_1) + 1/(4\zeta_2) - 1]^2$ $(\approx M_{B-L}^4[1/(4\zeta) - 1]^2)$, where $\zeta_1 \equiv \rho_1 M_{B-L}^2/(\kappa_1 M_*^2)$, and $\zeta_2 \equiv \rho_2 M_{B-L}^2/(\kappa_2 M_*^2)$. Equations (19)–(22) are valid only when $M_{B-L}^2|1/(4\zeta) - 1|/2 \lesssim |\langle S\rangle|^2 << M_P^2$.[6] In the limit $M_P \to \infty$, the above results approach the values in global SUSY [10].

Since P and Q develop VEVs, $X^{(\prime)}$, Y, $\mathbf{45}_H$, and $\mathbf{45}_H'$ should also achieve VEVs from $D_{\mathbf{16}^{(\prime)}}W = D_{\overline{\mathbf{16}}^{(\prime)}}W = 0$ even during inflation. Consequently, $SO(10)$ and $U(1)_A$ are broken to $G_{\rm SM}$ during inflation. Note that $\langle P\rangle = \langle \overline{P}\rangle$ and $\langle Q\rangle = \langle \overline{Q}\rangle$ in Eqs. (20) and (21) lead to $\langle P_-\rangle = \langle Q_-\rangle = 0$. A nonzero vacuum energy from the F-term potential induces universal "Hubble induced scalar mass terms" $(\kappa^2 M_0^4/M_P^2 \times |\phi_l|^2)$, which are read off from Eq. (15). But such small masses $(\kappa M_0^2/M_P << M_{B-L})$ cannot much affect the VEVs of the superheavy scalars of order $M_{\rm GUT}$.

Indeed, as seen earlier, in the SUSY limit the VEVs of P, $\overline{P}$, Q, $\overline{Q}$ are not determined, even though $\langle P\overline{P}\rangle$ and $\langle Q\overline{Q}\rangle$ are fixed. But by including the SUSY breaking soft terms of order $m_{3/2}$ in the scalar potential, they are completely determined. Thus, one might expect that the nonvanishing VEV of S and the "Hubble induced masses" $(>> m_{3/2})$ during inflation cause the VEVs of P, $\overline{P}$, Q, $\overline{Q}$ to significantly deviate from their values at low energies. Such differences, if true, would result in oscillations by P, $\overline{P}$, Q, and $\overline{Q}$ (or $P_\pm$ and $Q_\pm$) after inflation. As explained earlier, with universal Hubble induced masses, $\langle P_-\rangle = \langle Q_-\rangle = 0$. Since the VEVs of P_- and Q_- vanish both during and after inflation, oscillations by such light $(\sim m_{3/2})$ scalars would not arise after inflation has ended.

A mass term for S is induced by SUGRA corrections, such that the F-term potential contains

[6] $\mathbf{16}_H$ and $\overline{\mathbf{16}}_H$ develop the VEVs in the neutrino directions $\langle \nu_H^c\rangle$, $\langle \bar\nu_H^c\rangle$. Near the VEVs during inflation, the normalized real scalar fields, ${\rm Re}(\delta\nu_H^c + \delta\bar\nu_H^c)$ and ${\rm Im}(\delta\nu_H^c - \delta\bar\nu_H^c)$, acquire mass squareds given by $m_\pm^2 \approx 4\kappa^2|\langle S\rangle|^2 \pm 2\kappa^2 M_{B-L}^2[1/(4\zeta) - 1]$, respectively [10].

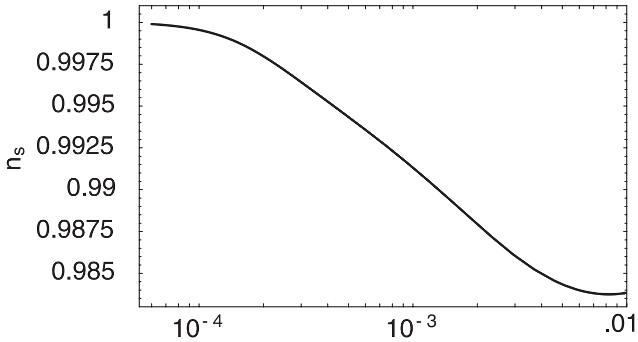

FIG. 1. The spectral index n_s vs κ. κ is <0.01 so that the reheat temperature does not exceed 10^9 GeV. See Fig. 3.

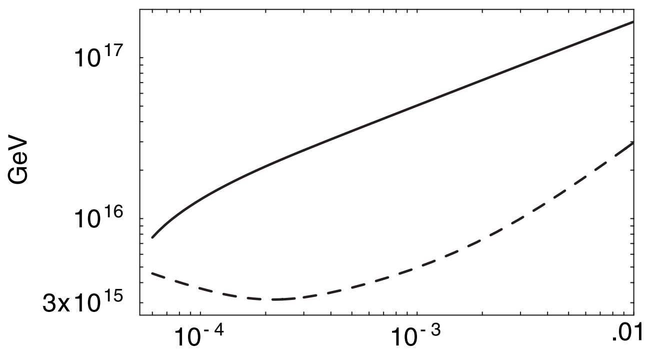

FIG. 2. The symmetry breaking scale M_{B-L} (solid line) and magnitude of the inflaton $|S|$ (dashed line) vs κ.

$$V_F \supset \sum_l |D_{\phi_l} W|^2 \sim \left(\frac{M_{\rm GUT}}{M_P}\right)^2 \times H^2 |S|^2, \qquad (23)$$

where H ($\equiv \kappa M_{\rm eff}^2/M_P \approx \kappa M_0^2/M_P$) denotes the Hubble induced mass. Such a small mass term of S ($< < H^2|S|^2$) does not spoil the slow-roll conditions. The correction term in Eq. (23) has a small impact on the inflationary predictions.

With SUSY broken during inflation ($F_S \neq 0$), there are radiative corrections from the $\mathbf{16}_H$, $\overline{\mathbf{16}}_H$ supermultiplets, which provide logarithmic corrections to the tree-level potential $V_F \approx \kappa^2 M_0^4$, and thereby drive inflation [1]. In our model, the scalar spectral index turns out to be $n_s = 0.99 \pm 0.01$ for $\kappa < 10^{-2}$. (See Fig. 1.) The symmetry breaking scale M_{B-L} is estimated to be around 10^{16}–10^{17} GeV (Fig. 2).

Before concluding, some remarks about the reheat temperature T_r, leptogenesis, and right-handed neutrino masses are in order. When inflation is over, the inflatons decay into right-handed neutrinos. Following Refs. [4,29], a lower bound on T_r is obtained for $\kappa = \lambda$, and the results are summarized in Fig. 3. (To obtain Figs. 1–3, we set $M_* = M_P$ and $\rho = \rho_{1,2} = 1 > > \kappa_{1,2}^2/\kappa^2$.) We see that $T_r \lesssim 10^9$ GeV for $\kappa \lesssim 10^{-2}$. The inflaton decay into right-handed neutrinos yields the observed baryon asymmetry via leptogenesis. Assuming nonthermal leptogenesis and hierarchical right-handed neutrinos, we estimate the three right-handed neutrinos' masses to be of order 10^{14} GeV, (10–20) $\times\, T_r$, and few $\times\, T_r$. Note that with $\kappa < 10^{-2}$ the inflaton (with mass $\sim\sqrt{\kappa M_{B-L}^2}$) cannot decay into the heaviest right-handed neutrino (of mass $\sim 10^{14}$ GeV). Thus, the latter does not play a direct role in leptogenesis.

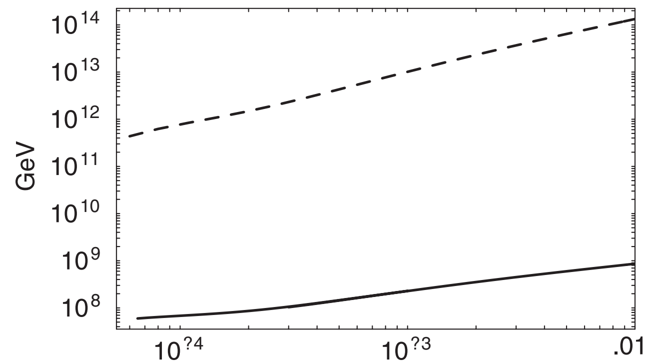

FIG. 3. Reheat temperature T_r and inflaton mass (dashed line) vs κ.

In summary, our goal here was to realize inflation in a realistic SUSY $SO(10)$ model. A global $U(1)_A$ and the $U(1)$ R symmetry play essential roles in the analysis. Several testable predictions emerge. In particular, the scalar spectral index $n_s = 0.99 \pm 0.01$, which will be tested by several ongoing experiments. Proton decay proceeds via $e^+\pi^0$, with an estimated lifetime of order 10^{34}–10^{36} yrs. The LSP is stable and the MSSM parameter $\tan\beta$ is large, of order m_t/m_b. Two of the three right-handed neutrino masses are fairly well determined. The heaviest one weighs around 10^{14} GeV, and the one primarily responsible for nonthermal leptogenesis has mass of order 10 T_r, where the reheat temperature T_r is around 10^8–10^9 GeV.

We thank Nefer Senoguz for helpful discussions and for providing us with the figures. This work is partially supported by the DOE under Contract No. DE-FG02-91ER40626 (Q. S.).

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